WIRELESS ENGINEER

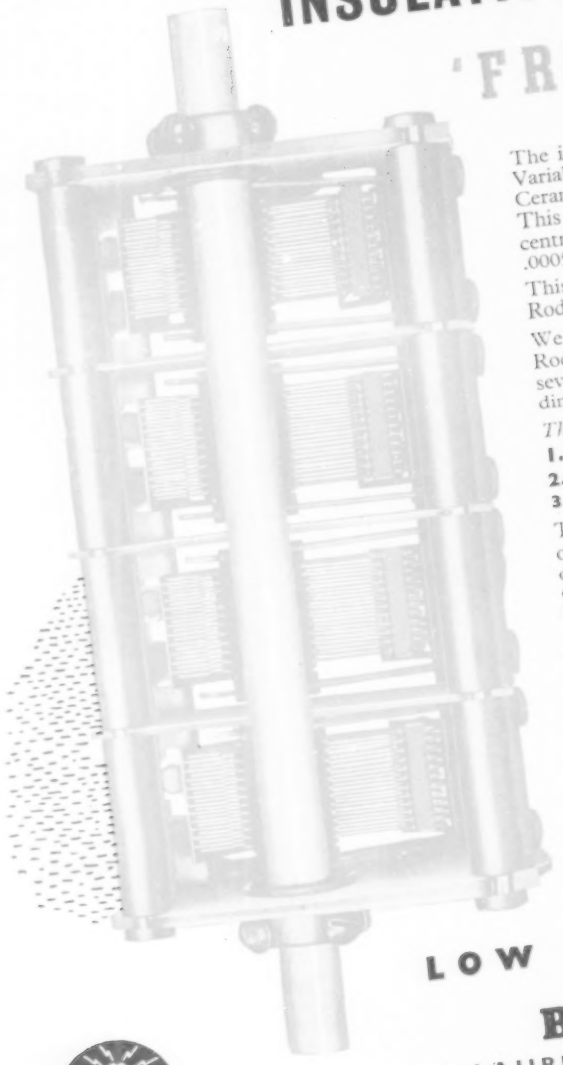
THE JOURNAL OF RADIO RESEARCH & PROGRESS

MARCH 1954

VOL. 31 No. 3 SIX SHILLINGS

Including Index to Vol. 30 and Index to Abstracts 1953
(formerly published separately)

FOR HIGH-FREQUENCY INSULATION—specify 'FREQUELEX'



The illustration shows a Four Gang Radio Variable Condenser using our "**FREQUELEX**" Ceramic Rod for the Centre Rotating Spindle. This Rod is 7½″ long × .437″ diameter, centreless ground to within plus or minus .0005″. Maximum camber allowance of .002″. This is only one of many applications where Rods made to close limits are required.

We specialise in the manufacture of Ceramic Rods and Tubes of various sections in several classes of materials over wide dimensional ranges.

The Principal Materials Are :—

1. Porcelain for general insulation.
2. Frequelex for High Frequency insulation.
3. Permalex and Templex for Capacitors.

The degree of accuracy depends on the size of the Rod or Tube, but the standard degree of accuracy is outlined in the Inter Service Component Manufacturer's Council—Panel R Specification embodied in our Catalogue of Radio Frequency Ceramics, copy of which will be sent on request.

Large Rods up to 44″ long and 1¼″ square are used as supports for Tuning Coils, etc.

We shall be pleased to have your enquiries for all sizes of Tubes and Rods. Prompt deliveries can be given for most sizes.

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'VARIAC' voltage regulating transformers

Reg'd Trade Mark

With a 'VARIAC' voltages are instantly and minutely adjustable from 0-Line Voltage, or in some cases up to 17% above line voltage. Type 50-B 'VARIAC,' as illustrated left, is often operated in a 3-gang assembly on 3-phase work to control 21kVA.

Type 200 C.U.H. 'VARIAC'

Type 100-R 'VARIAC'

SERIES 50 'VARIAC' TRANSFORMERS.

SPECIFICATIONS

Type	Load Rating	Input Voltage	Current		Output Voltage	No-Load Loss	Net Price £ s. d.*
			Rated	Maximum			
50-A	5 kva.	115 v.	40 a.	45 a.	0-135 v.	65 watts	44 18 6
50-B	7 kva.	230/115 v.	20 a.	31 a.	0-270 v.	90 watts	44 18 6

All 'VARIAC' prices plus 20% as from 23rd Feb. 1952

REQUEST ALSO OUR 20-PAGE SUPPLEMENTARY CATALOGUE GIVING COMPLETE INFORMATION ON OUR NEW AND COMPLETE RANGE OF AC AUTOMATIC VOLTAGE STABILISERS: THESE RANGE FROM 200 VA TO 25 kVA. PERFORMANCE IS EXCELLENT, FROM NO-LOAD TO FULL LOAD, AND STABILITY IS QUITE UNAFFECTED BY FREQUENCY VARIATIONS.

PRICES ARE EXCEPTIONALLY LOW.

Write for catalogue V549 which gives full details of 'VARIAC' transformers and suggestions for use.

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OC10 OC11 OC12 — 3 JUNCTION TRANSISTORS for circuit experiments

Three types of junction transistor, the Mullard OC10, OC11 and OC12 are now available for circuit experiments.

In the past, the lack of supplies has prevented circuit designers in this country from gaining direct experience of junction transistors in their own laboratories. Now, however, the availability of the first junction types invites practical investigation into their many possible applications.

As junction transistors provide no current gain when connected with grounded base, they are more usually employed in grounded emitter circuits, where they function well as A.F. amplifiers. In both amplifier and oscillator circuits these transistors will operate with supply voltages as low as 1.5 V and with current consumptions of the same remarkably low order.

The OC11 is a general-purpose amplifier, while the OC12 is intended for operation in an output stage, although it can, of course, be used otherwise. A low-noise version of the OC11 is provided by the OC10, a special transistor for early stages in high-gain amplifiers.

Junction transistor type		OC10	OC11	OC12
Max. D.C. negative collector-to-emitter voltage	(V)	4	4	4
Typical D.C collector voltage	(V)	2	2	2
Typical collector current	(mA)	−0.5	−0.5	−2
Current amplification factor (α') with grounded emitter		17	17	30
Output resistance with infinite A.C. source impedance (grounded base)	(KΩ)	700	700	500
Special low-noise characteristics		★	—	—

★ Superior type for these characteristics.

Information on these junction transistors and the point-contact types in the Mullard range of semi-conducting devices will be gladly supplied by the Industrial Technical Service Department at the address below.

● The OC10, OC11 and OC12 are readily available for experimental purposes at a price comparable with that of mains subminiature valves.

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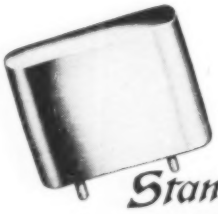
Available with accuracies better than 50 cycles in one megacycle; artificially aged, gold plated and wire mounted **STC** crystals greatly reduce drift under working conditions. Moreover, they will maintain this high degree of accuracy under severe vibration and sharply changing climatic conditions.



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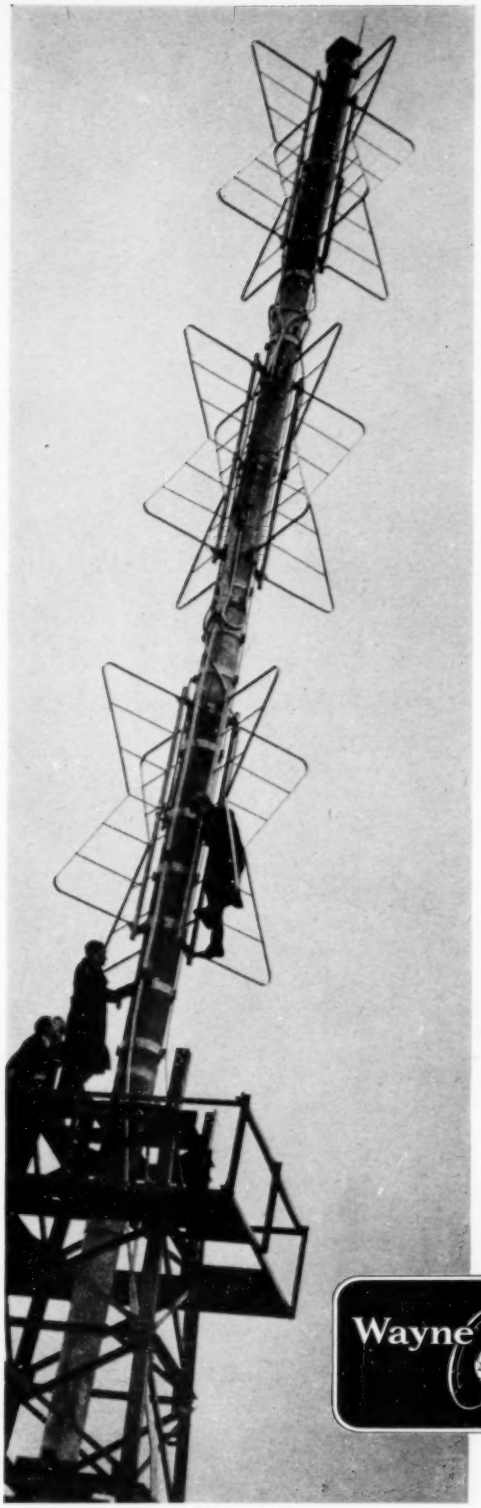
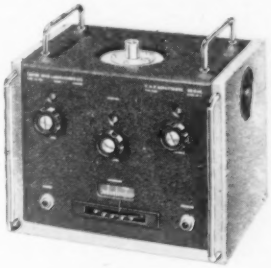


Broad-band Matching

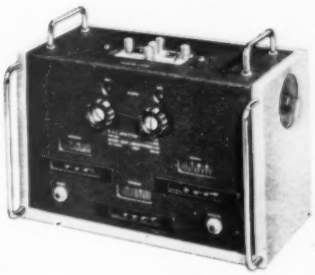
The photograph shows Marconi engineers erecting for test the three-stack super-turnstile TV aerial for the new B.B.C. transmitter at Pontop Pike. Wayne Kerr Bridges are used for matching feeders and transmission lines to the radiators.



FOR UNBALANCED MEASUREMENT FROM 50—250 MC/S

B.901

Susceptance: Equivalent to ± 75 pF to ± 2%, ± 0.5 pF
Conductance: 0-100 mmho to ± 2%, ± 0.1 mmho



FOR BALANCED AND UNBALANCED MEASUREMENT FROM 1—100 MC/S

B.801 and B.701

Susceptance: Equivalent to ± 230 pF to ± 2%, ± 0.5 pF
Conductance: 0-100 mmho to ± 2%, ± 0.1 mmho
Susceptance: Equivalent to ± 80 pF to ± 2%, ± 0.5 pF
Conductance: 0-100 mmho to ± 2%, ± 0.02 mmho

FOR BALANCED AND UNBALANCED MEASUREMENT FROM 15 KC/S—5 MC/S

B.601

Capacitance: 0.01 pF — 20,000 pF
Resistance: 10 ohms — 10 megohms
Inductance: 0.5 μH — 50 mH
Accuracy: 1% over major part of range

These Wayne Kerr Bridges are used with external source and detector for the measurement of aerials, cables, feeders, and a variety of components and materials.

Photograph by courtesy of Marconi's Wireless Telegraph Co. Ltd.

Wayne Kerr

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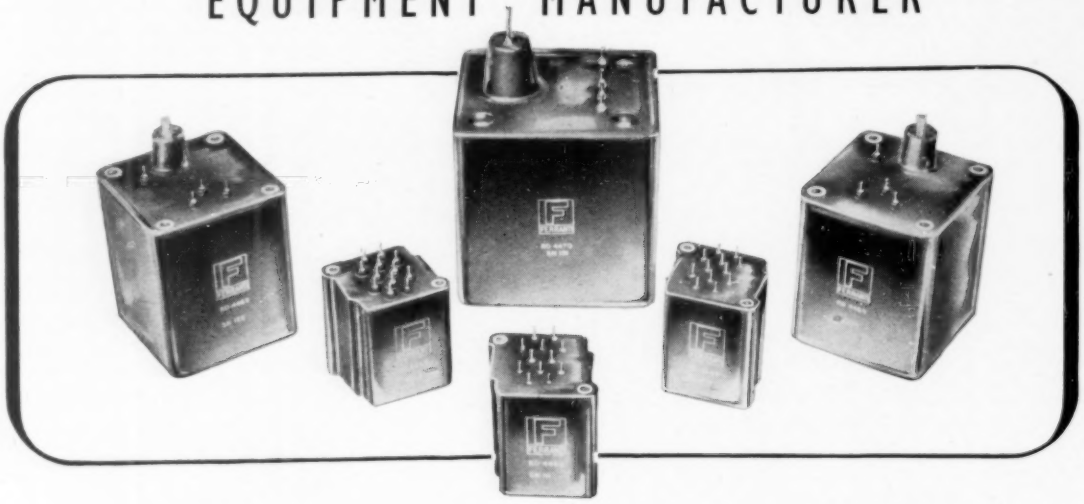

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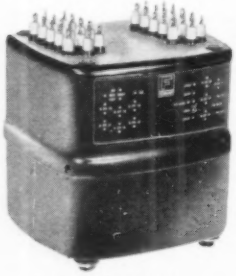

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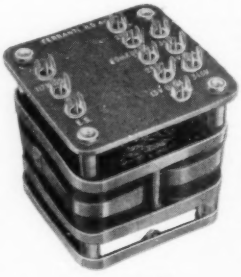
hermetically sealed oil-filled components. "C" core construction, conforming with RCL.215.



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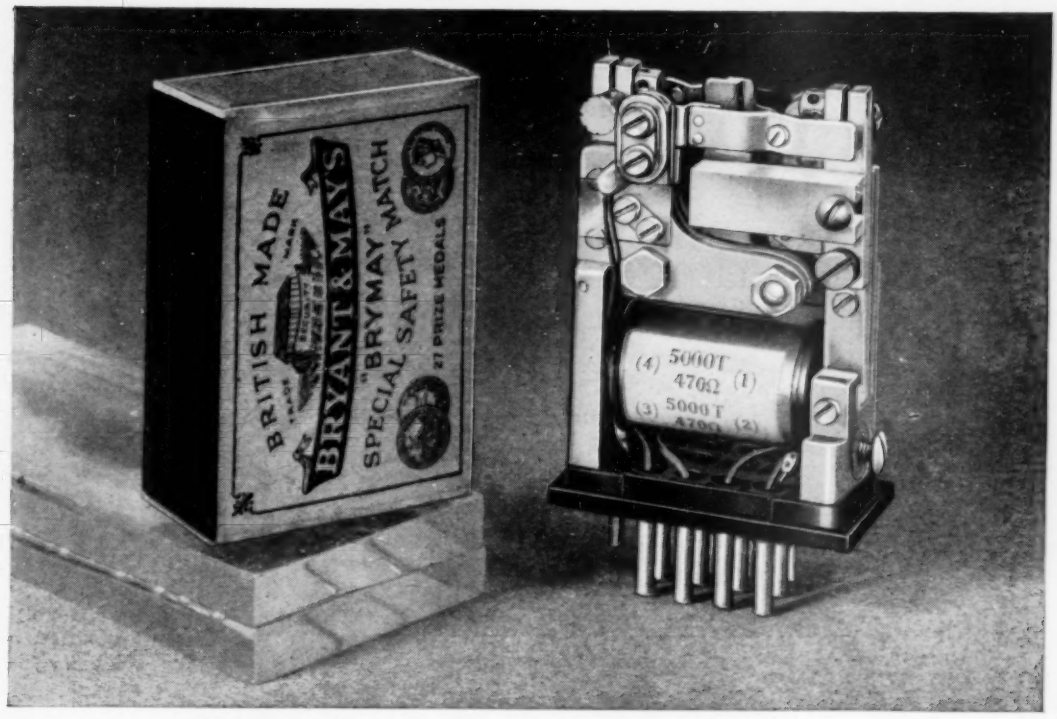

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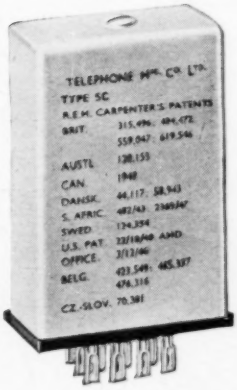
Considering it is only the size of a matchbox the Type 5 Carpenter Polarized Relay is capable of a surprisingly high performance providing answers to problems in many fields of electrical engineering. Outstanding features of the Type 5 include:

HIGH OPERATIONAL SPEED • FREEDOM FROM CONTACT REBOUND
IMMUNITY FROM POSITIONAL ERROR • GOOD CONTACT PRESSURES
HIGH SENSITIVITY • ACCURACY OF SIGNAL REPETITION
RUGGED DESIGN • EXCEPTIONAL THERMAL STABILITY



Plug or solder tag base optional.

Dimensions—(*With cover. Excluding connecting pins.*) $2\frac{3}{16}$ *ins. high.* $1\frac{7}{16}$ *ins. wide.* $\frac{3}{4}$ *in. deep. Weight* (*including socket*) 4.8 *oz.* (137 *gm.*)

Complete specification and further details of the complete range of Carpenter Relays may be had on request.

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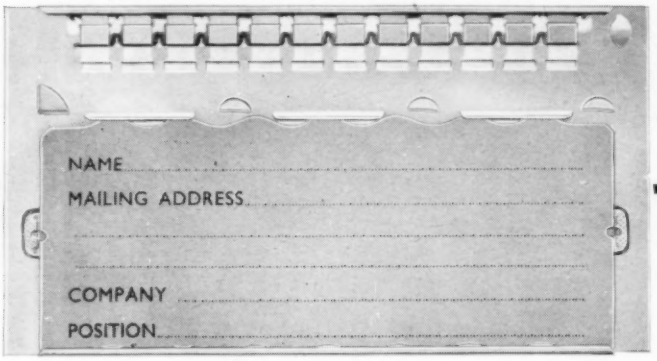

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MUREX 'SINCOMAX' MAGNETS

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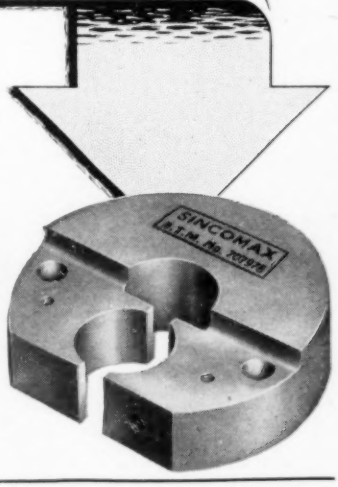

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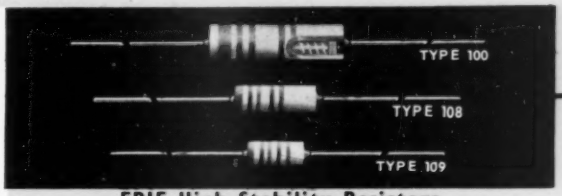
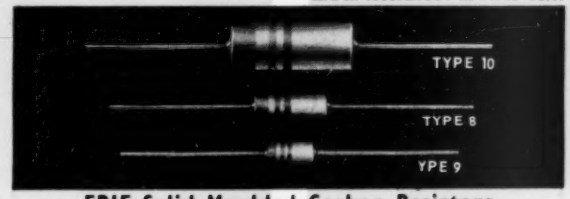
There is a wide selection of disc and moulded Ceramicons* for various applications, covering voltages up to a maximum of 30 kV.



ERIE High Stability Resistors

The only resistor of this class in which the supersensitive carbon film is ceramic encased. Available in ratings of ¼ watt, ½ watt and 1 watt, in values ranging from 10 ohms to 3 megohms, and in tolerances down to ±1%.



ERIE Solid Moulded Carbon Resistors

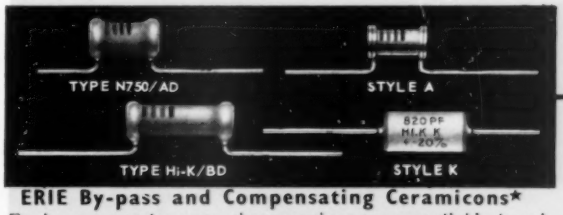
Available in ratings of ¼ watt, ½ watt and 1 watt, either phenolic or ceramic insulated, in values ranging from 10 ohms to 10 megohms, and in tolerances down to ±5%.

ERIE*

dependable electronic components

ERIE Disc Ceramicons*

Available in values ranging from 2.5 PF to 30,000 PF in working voltages from 500 to 8 kV, and in tolerances down to ±10%. Capacity variations with temperature, age, and voltage are exceptionally small. A truly outstanding range.



ERIE By-pass and Compensating Ceramicons*

For by-pass requirements, there are three types available in values ranging from 220 to 18,500 PF. For compensating, there are nine types in temperature coefficients ranging from P100 to N750, available in values up to 1480 PF.

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The largest and most versatile range of plastic and ceramic temperature compensating trimmer capacitors. Available in values ranging from 0.5 PF minimum to 30 PF maximum.

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Specially designed to overcome radiation and critical by-passing problems. Available in values up to 1500 PF.

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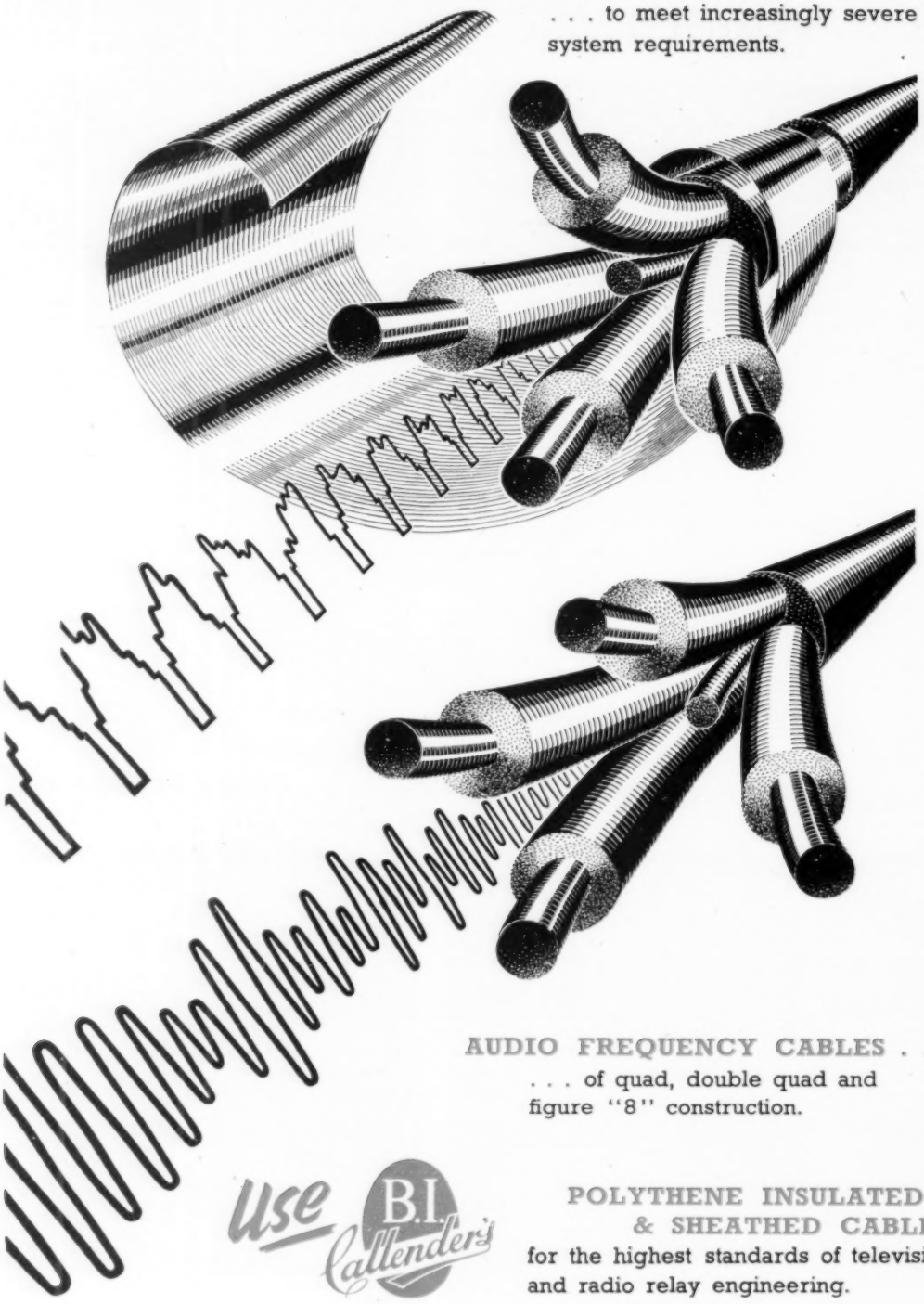

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constant voltage transformer . . .

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CONSTANT VOLTAGE
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Input 230 *volts, nominal* (190/260 *volts*) 50 *c/s.*
Output 230 *volts, plus or minus* 1%.

Type CVH 125 — *maximum load* 125 *watts, weight* 20 *lb.*
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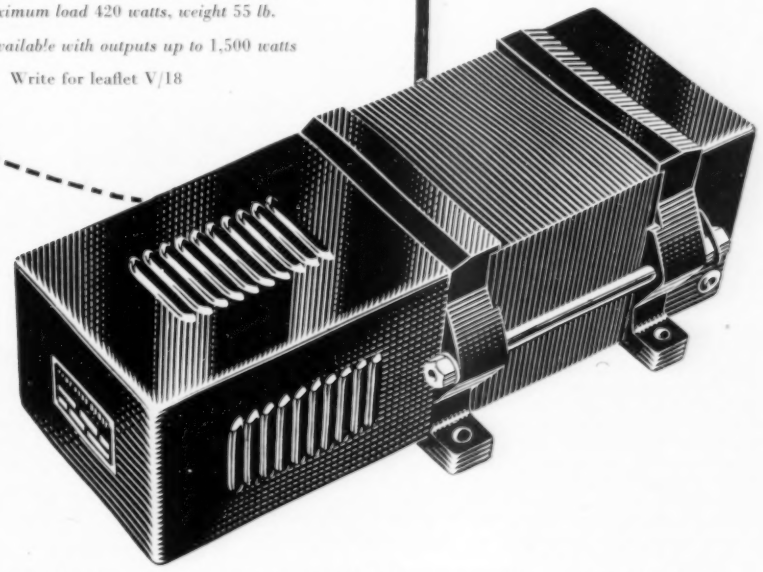
Other models available with outputs up to 1,500 *watts*

Write for leaflet V/18



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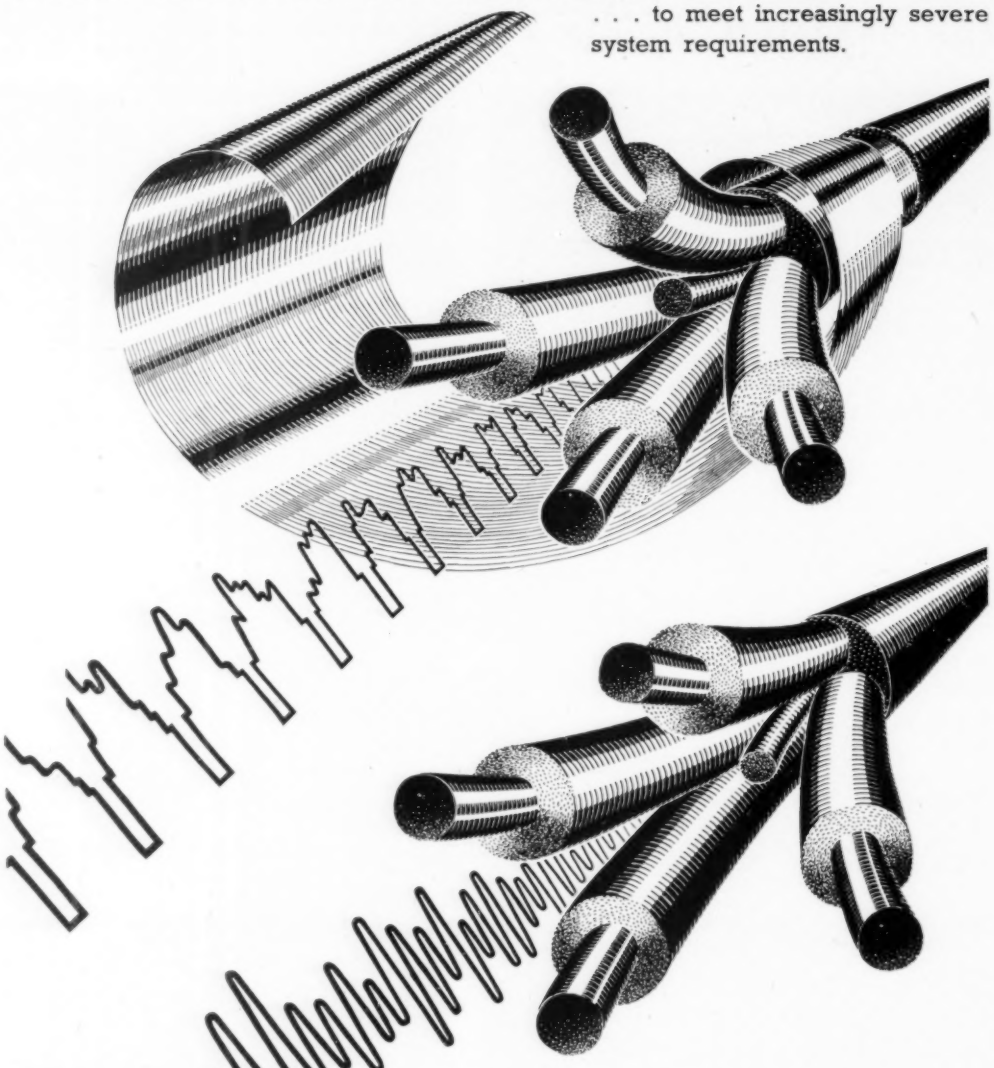

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Advance
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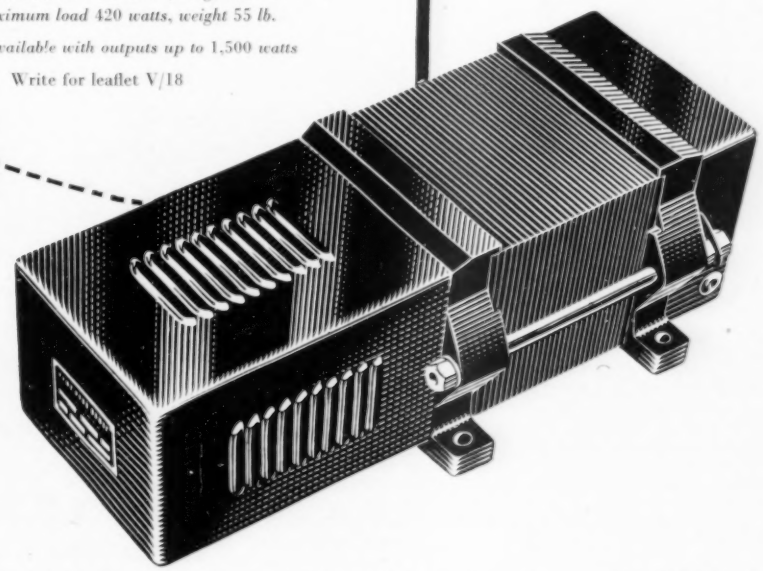
PROVIDES
CONSTANT VOLTAGE
WITH
REDUCED HARMONIC
DISTORTION

Input 230 *volts, nominal* (190/260 *volts*) 50 *c/s.*
Output 230 *volts, plus or minus* 1%.

Type CVH 125 — *maximum load* 125 *watts, weight* 20 *lb.*
Type CVH 420 — *maximum load* 420 *watts, weight* 55 *lb.*

Other models available with outputs up to 1,500 *watts*

Write for leaflet V/18



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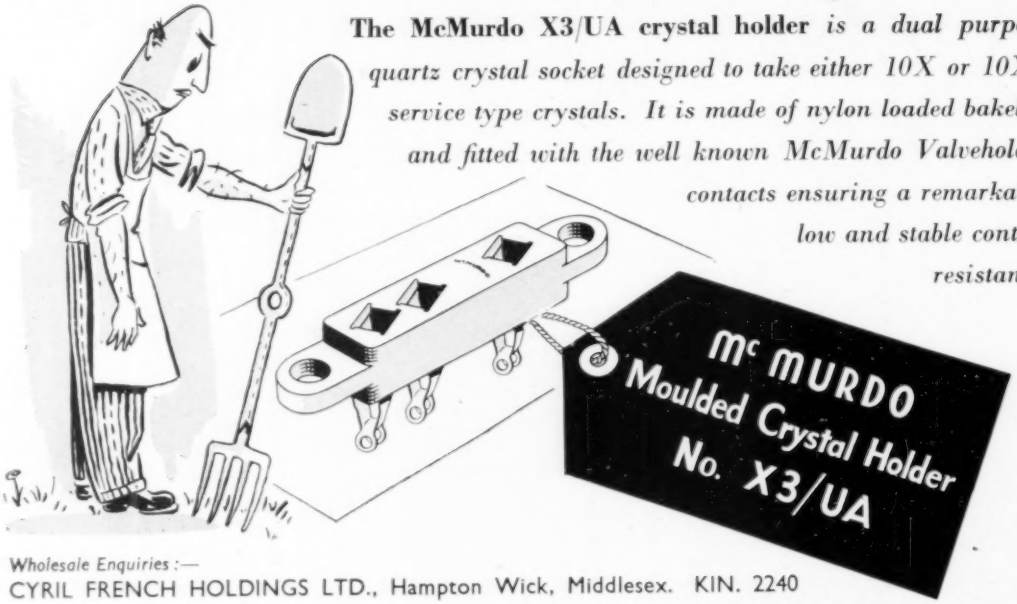

The McMurdo X3/UA crystal holder *is a dual purpose quartz crystal socket designed to take either 10X or 10XJ service type crystals. It is made of nylon loaded bakelite and fitted with the well known McMurdo Valveholder contacts ensuring a remarkably low and stable contact resistance.*

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A Guide to Theory and Design

★ 2nd Edition

By Alan Douglas. The new edition of this comprehensive handbook covers every design phase of the modern electronic musical instrument. With the exception of "freak" or novelty devices, there is no electronic musical instrument in production which does not employ basically one or other of the circuits now shown in this edition. The book explains the relationship between electrical tone colours and their acoustic counterparts, and describes the most modern circuits for achieving satisfactory results. Profusely illustrated. 30/- net.

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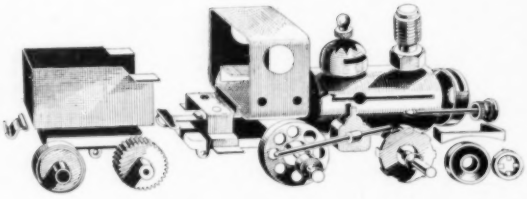

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ANTENNA RADIATION PATTERNS

THIS equipment has been developed by EKCO Electronics to automatically record the radiation patterns of any centimetric antenna. The antenna under test is mounted on the roof of a rotatable trailer and illuminated by a fixed transmitter. The amplitude of the received signal is then continuously plotted against the angular traverse of the trailer.

ALL the equipment except the transmitter unit is mounted in the trailer and remote controls for the transmitter are provided. The received C.W. signal is mixed with a modulated Local Oscillator signal and the resultant I.F. output combined with an anti-phase modulated I.F. signal. The reference signal is derived from a 30 Mc/s Oscillator and Servo-driven Piston Attenuator. The combined signals are fed via a seven-stage, low-noise I.F. Amplifier to a balanced Modulator, and the resultant error signal applied to a Servo Amplifier. The output of this Amplifier drives a Servo Motor which moves the Piston Attenuator in such a direction as to reduce the difference between the reference signal and the received signal. A pen attached to the piston drive mechanism records the amplitude of the received signal in terms of the attenuation law of the standard piston.

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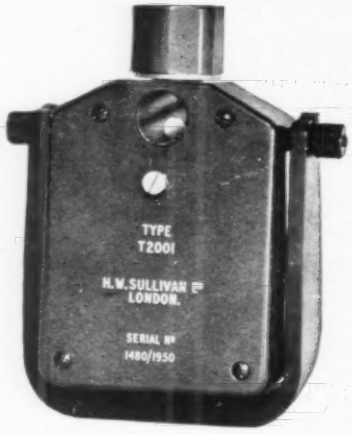

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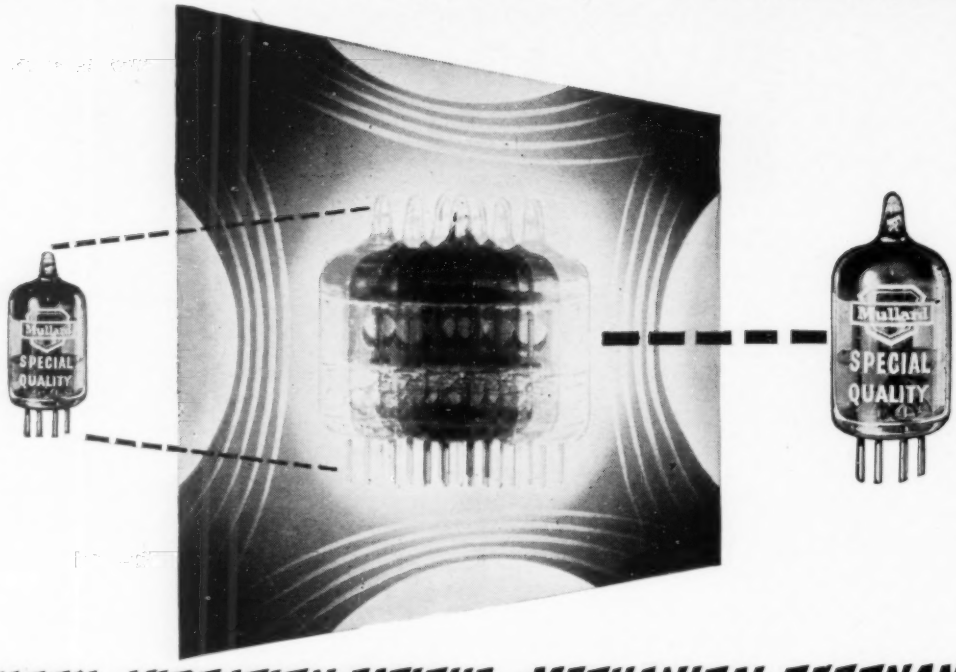

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WIRELESS ENGINEER

Vol. 31 MARCH 1954 No. 3

Edwin H. Armstrong

By his death on 1st February the United States lost one of the outstanding personalities of the wireless world. Edwin Howard Armstrong was born in New York on 18th December 1890. He graduated in Electrical Engineering at Columbia University in 1913 and was awarded an honorary D.Sc. in 1929. He was an assistant in the electrical engineering department for a year after graduation, and then for 21 years collaborated with Michael Pupin in research at the Marcellus Hartley Research Laboratories in Columbia University. From 1934 until his death he was a Professor of Electrical Engineering in the University. During the first world war he spent two years, first as captain and then as major, in the Signal Corps, and in 1919 was made a Chevalier of the Légion d'Honneur.

The name of Armstrong is most closely associated with four inventions, viz. the regenerative circuit, 1912; the superheterodyne system of reception, 1917-18; the superregenerative circuit, 1920; and frequency modulation, 1935. He was a lad of 16 when in 1906 Lee de Forest patented the 3-electrode valve, which became known as the audion, and it was in 1912, while still a student at Columbia, that he was experimenting with an audion when he discovered the presence of h.f. current in the anode circuit, which led to his invention of regeneration and the valve oscillator, and endless patent litigation. In 1914 he published a correct explanation of the action of a triode and disproved some of the currently-accepted ideas; in 1915 he read a paper on regeneration before the Institute of Radio Engineers, and in 1916 another paper on the heterodyne detector. The impact of these papers may be judged from the fact that in the following year the Institute of Radio Engineers awarded him the first Medal of Honour for his work on regeneration and the production of oscillations. Seventeen years later there was a somewhat tragic sequel to this award for, following the adverse decision of the U.S. Supreme Court on the question of priority of invention of these discoveries, he returned the medal to the Institute in 1934. The Board of Directors, however, unanimously declined to accept it and reaffirmed the original award.

Though naturally not entirely unbiased, some light is thrown on this long drawn-out litigation by the autobiography of Lee de Forest, published in 1950. At the same time as Armstrong was experimenting with the audion in New York, Lee de Forest and two assistants were working on somewhat similar lines at Palo Alto in California, and the fight as to who was the prior inventor went on for 20 years.

In the autumn of 1913 de Forest read a paper on "The Audion Amplifier" before the I.R.E. at Columbia University and he says: "My demonstration of the crashing sounds emitted from my loudspeaker when I dropped a handkerchief on the table before the telephone receiver serving as my 'pick-up' aroused great astonishment and applause. On that occasion young Edwin H. Armstrong, wrapped in deepest mystery, had a small carefully-concealed box in an adjoining room into which neither I nor my assistant Logwood was permitted to peek. But when he led two wires to my amplifier input to demonstrate

the squeals and whistles and signals he was receiving from some transmitter down the Bay, we thought we had a pretty fair idea of what the young inventor had concealed in his box of mystery. So we proceeded, meekly and obediently, to amplify whatever signals came over the wires from that room".

That is the first mention of Armstrong in de Forest's autobiography, but early in 1914 de Forest demonstrated his ultra-audion oscillator at the Bureau of Standards in Washington, and he says that Professor Pupin, whom he had long known as a kindly friend, loudly demanded "What right have you to have that here? That thing is not yours. That belongs to Armstrong." He says that he was too flabbergasted to reply, but gazed upon his surprising wrath and "continued the siren sounds." He proceeds, "Then I knew for a certainty what it was that Armstrong had had in his little magic box at Columbia. And that outburst by Professor Pupin was the opening gun of the bitterly contested patent battle to be waged for years in the Patent Office interference proceedings; and thereafter for years more until at long last the U.S. Supreme Court should finally decide the historic contest." Later de Forest says: "On January 15, 1920, I read my paper on the Audion and its evolution before the Franklin Institute at Philadelphia. It was well received, except by one E. H. Armstrong, who sought to show that it was he who had invented the feed-back circuit. 'All de Forest invented was the Audion! We'll concede that', he growled. Whereupon the chairman ordered him to sit down."

The feedback patent, which, after nearly 20 years' litigation, was finally awarded to de Forest, expired in 1941. It had been in turn awarded to Armstrong, then Langmuir, then again to Armstrong, and finally to de Forest. One can appreciate the feelings that prompted Armstrong to return the medal to the Institute.

Another of Armstrong's inventions, with much happier associations, is frequency modulation. This occurred to him as the result of some experiments he and Pupin made with the idea of eliminating static interference; experiments which, he says, were unsuccessful, but which laid the foundations of his system of reducing disturbance by using frequency modulation. In an outline of the history of f.m., which he gave before a section of the I.R.E. in 1946, he said that he started looking for a static eliminator back about 1914, and that he worked a little longer than most people did. He then hit upon the idea of frequency-shift keying and from that went on to frequency modulation. It is pleasing to note that towards the end of his autobiography de Forest says: "Major E. H. Armstrong deserves the greatest credit for the development of his system of frequency-modulation—brought out in spite of the skepticism of the profession, and a reluctant Federal Communications Commission. He has given to radio broadcasting a new arm; for this I salute him."

In 1935 the Radio Club of America founded a medal to be known as the Armstrong Medal. In 1941 Armstrong was awarded the Franklin Medal by the Franklin Institute, and in 1943 the Edison Medal by the American I.E.E.; he was also awarded medals by many other institutions.

In 1947 he received a Medal of Merit and a Presidential Citation for his contributions to military radio communications.

In "Radio: Beam and Broadcast", by A. H. Morse, published in 1925, the patent litigation up to that time is discussed very fully: the author concludes by saying that, "Armstrong's work in radio is such that, had he no patented or patentable inventions—and he has many—he would still rank as one of the foremost exponents of the art". This was before the invention of frequency modulation.

Since 1948 he had been working on what he called the multiplexed transmission of frequency-modulated signals, and as recently as last October he and J. S. Bose, also of Columbia University, read a paper on the subject and gave demonstrations before the Radio Club of America. The multiplex system enables two programmes to be broadcast simultaneously within the standard f.m. band of 200 kc/s. Earlier attempts at multiplexing were not very successful because of cross-modulation between the main and the auxiliary channel and of noise transfer from one to the other, but as the result of five years of work they claimed to have overcome the difficulties, and to be able to obtain results on their second or auxiliary channel superior to those obtained by ordinary amplitude-modulated stations.

One can only regret that so much of his life was overshadowed and embittered by such protracted patent litigation, but during the last 18 years he had the great satisfaction of seeing his frequency modulation becoming more and more highly appreciated, and replacing amplitude modulation on an increasing scale. His name will ever be associated with this outstanding achievement.

G. W. O. H.

HIGH-Q COUPLED TUNED CIRCUITS

By Haim D. Polishuk, M.Sc.

SUMMARY.—By means of a set of expressions developed as functions of a frequency ratio, some aspects of inductively-coupled high-Q resonant circuits are analysed. Considerations of finite, very high-Q circuits lead to simple relations determining impedance characteristics of a fundamental system, resonant frequencies, rate of frequency deviation, input conductance, stored energy and power dissipation ratios.

1. Introduction

THE general subject of inductively-coupled resonant circuits has been treated by many writers and explored with exhaustive rigour. However, present advances in practical analysis of electronic systems, and in particular those related to the u.h.f. domain, suggest an attempt at a somewhat different approach to the investigation of the performance of high-Q coupled tuned circuits. These circuits, apart from constituting the basic frequency-selective communication elements, appear now fundamental in the study of the physical characteristics of an increasing number of u.h.f. applications. For example, electron-tube systems designed for frequency modulation, stabilization and control, where coupled resonant cavities are predominantly employed and for which information of an essentially different nature is sought.

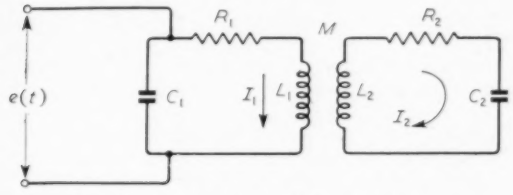

Fig. 1. *Inductively-coupled tuned circuits.*

A fundamental treatment is presented here,* which is also applicable, in many respects, to lower-frequency systems, in which a set of parameters is analytically developed and examined, with the aim of predicting in a general and practical way the steady-state behaviour (excluding mode phenomena) of tightly-coupled high-Q resonant circuits. In expounding the aspects of such circuits a derived group of simple compact expressions is made to determine such information as: effects of tuned-circuit coupling, resultant general resonant frequencies, power dissipation ratio, stored-energy ratio, frequency-deviation rate, resonant conductance characteristics, system effective Q, etc.

MS accepted by the Editor, May 1953

* In a way, an outgrowth of some of the ideas introduced in the works of Sherman, Tellegen, Cuccia and others. See References.

Throughout the following discussion, repetition of elementary or well-known principles has been purposely avoided, except where it is warranted by giving a clearer interpretation of the subject matter.

2. Characteristics of Coupled Impedance

Fig. 1 represents a generalized circuit of an inductively-coupled two-mesh network, excited by a voltage $e(t) = E_0 e^{j\omega_0 t}$, to which a considerable variety of practical systems can be reduced.

Analysis has shown that the series impedance reflected into the primary circuit from the secondary is

$$Z_k = \frac{-(j\omega M)^2}{R_2 + j(\omega L_2 - 1/\omega C_2)} \quad \ldots \quad \ldots \quad (1)$$

It should be remembered that this expression of the coupled impedance would not change if, instead of the primary being a tank circuit, a series resonant circuit (C_1 in series with R_1 and L_1) were considered.

The following basic definitions will be used:

ω = $2\pi \times$ frequency

ω_1 = resonant angular frequency of isolated primary circuit $= (L_1C_1)^{-\frac{1}{2}}$

ω_2 = resonant angular frequency of isolated secondary circuit $= (L_2C_2)^{-\frac{1}{2}}$

$Q_1 = \omega_1 L_1/R_1 = 1/\omega_1 C_1 R_1$

$Q_2 = \omega_2 L_2/R_2 = 1/\omega_2 C_2 R_2$

k = effective coefficient of coupling $= M(L_1L_2)^{-\frac{1}{2}}$.

To express Z_k as a function of a frequency ratio, let

$$t = \omega/\omega_2 \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

then, substituting in (1),

$$Z_k = \frac{\omega^2 M^2}{R_2\left(1 + jQ_2\dfrac{t^2 - 1}{t}\right)} \quad \ldots \quad \ldots \quad (3)$$

which, after rationalizing and separating, may be put in the form:

$$Z_k = k^2 \frac{L_1}{L_2} R_2 \left(\frac{t^2}{1 - t^2}\right)^2 + jk^2\omega_2 L_1 \frac{t^3}{1 - t^2} \quad (4)$$

where a major simplification was rendered possible by introducing the practical assumption that Q_2 is of very large magnitude, or that

$$1/Q_2^2 \to 0 \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Considering Z_k as given by (4), it follows that the coupled reactance will remain inductive as long as $t^2 < 1$ or, for all values of $\omega < \omega_2$; whereas, when $t^2 > 1$, or for all values of $\omega > \omega_2$, it will appear capacitive. Moreover, it is noted that beyond the close vicinity of the singularity at $t = 1$, Z_k may be regarded as purely reactive. Fig. 2 reveals the general behaviour of the coupled resistance, R_k, and reactance, X_k, for varying values of t. It is further observed that the coupled reactance will display a maximum at $t = \sqrt{3}$ or, $\omega = \sqrt{3}\omega_2$, and will vary asymptotically between $-\infty$ and the line described by $X_k = -(k^2\omega_2 L_1)\,t$. Concurrently, R_k will vary between $+\infty$ and the line $R_k = k^2 R_2 L_1/L_2 =$ constant.

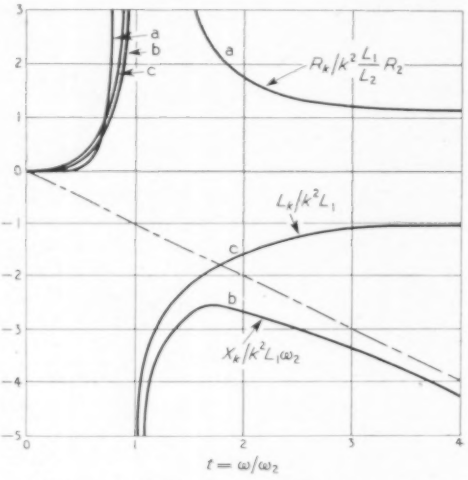

Fig. 2. *General behaviour of coupled resistance, reactance and effective inductance as a function of the ratio* ω/ω_2.

The effective coupled inductance may now be given directly by

$$L_k = \frac{X_k}{\omega} = L_1 k^2 \frac{t^2}{1 - t^2} \quad .. \quad .. \quad (6)$$

a plot of which, as a function of t, is also shown in Fig. 2.

In order to effect a more convenient change of frequency parameters, let us now consider the particular case where $\omega = \omega_0$, the general system resonant angular frequency, and let

$$s = \frac{\omega_0}{\omega_1} \text{ and } p = \frac{\omega_2}{\omega_1} \quad .. \quad .. \quad .. \quad (7)$$

Hence, $t = s/p$ and Z_k, as it is expressed in (4), may be transformed into

$$Z_k = R_1 \cdot \frac{p}{k^2}\frac{Q_1}{Q_2}\left(\frac{s^2 k^2}{s^2 - p^2}\right)^2 - j\omega_0 L_1 \cdot \left(\frac{s^2 k^2}{s^2 - p^2}\right) \quad .. \quad .. \quad .. \quad (8)$$

Putting

$$\beta = \frac{s^2 k^2}{s^2 - p^2} \quad .. \quad .. \quad .. \quad .. \quad (9)$$

and

$$\alpha = \frac{p}{k^2}\frac{Q_1}{Q_2}\beta^2 \quad .. \quad .. \quad .. \quad .. \quad (10)$$

it follows that the inductive branch of the primary circuit could be represented by the equivalent series impedance

$$Z_e = R_1(1 + \alpha) + j\omega_0 L_1(1 - \beta) \quad .. \quad (11)$$

It is thus evident that when $\omega = \omega_0$ the effect of coupling a secondary circuit is to increase the resistive component of the primary impedance by a fractional coefficient α and to reduce the primary inductance by a fractional coefficient β.

3. Resonant Frequencies

When the entire system, considered in the region where $kQ_2 \gg 1$, and reduced effectively to an equivalent circuit composed of impedance $1/j\omega C_1$ in parallel with Z_e, exhibits resonance, the general resonant angular frequency will take the form:

$$\omega_0 = 1/\sqrt{L_1(1 - \beta) \cdot C_1} = \omega_1/\sqrt{1 - \beta} \quad .. \quad .. \quad .. \quad (12)$$

or, $$s^2 = 1/(1 - \beta) \quad .. \quad .. \quad (13)$$

Although conditions justifying relation (12) are to be referred to later in this discussion, it may also be arrived at by starting with a series-resonant primary circuit, developing (by means of Kirchhoff's loop equations) the expression for the driving-point impedance and applying the general resonant condition when voltage is in phase with primary current. From (9) and (13), eliminating s and solving for β,

$$\beta = \frac{(p^2 - 1) \pm \sqrt{(p^2 - 1)^2 + 4k^2p^2}}{2p^2} \quad .. \quad (14)$$

Fig. 3 illustrates the general behaviour of β as a function of p and k, showing variations of both positive and negative roots of β. Considering limiting conditions, it appears that the positive root of β varies asymptotically between $\beta = +1$ and $\beta = k^2$ for all values of k, while the negative root varies between $\beta = 0$ and $\beta = -\infty$, approximating to the hyperbolic relation of $\beta = -p^{-2}$ for small values of p $(1 \gg p^2 > 0)$.

Knowing the value of β, s can be determined readily from (13); furthermore, by substituting (9) in (13), s is expressed directly in terms of p and k, as given by

$$s = \left[\frac{(p^2 + 1) \pm \sqrt{(p^2 - 1)^2 + 4k^2p^2}}{2(1 - k^2)}\right]^{\frac{1}{2}} \quad (15)$$

Evidently, therefore, knowledge of the isolated

frequencies $\omega_2/2\pi$ and $\omega_1/2\pi$ (and k) enables one to determine immediately the value of s and, consequently, the two possible values of $\omega_0/2\pi$ ($= s \times \omega_1/2\pi$), the resultant general (upper and lower) resonant frequencies.

The existence of both positive and negative values of β is supported by the reasoning that at frequencies below $\omega_2/2\pi$ and $\omega_1/2\pi$ the total reactance of the primary circuit is effectively capacitive; hence, the coupled reactance is rendered inductive by β appearing as a negative quantity ($s < 1$). At frequencies above $\omega_2/2\pi$ and $\omega_1/2\pi$, however, the total primary reactance is inductive and upon being balanced by the coupled capacitive reactance ($s > 1$ and positive β), the system will display resonance. It should be observed, moreover, that $1 - \beta$ is always a positive quantity. As a direct result from these conclusions, it follows further that ω_0 can never have a real value within the numerical range $\omega_1 - \omega_2$.

If ω_2 and ω_1 differ only by a very small fractional value such that their ratio may be written as

$$p = 1 \pm 2\Delta \quad .. \quad (16)$$

where, in the power series expansion of p, any but the first-power terms of Δ are negligibly small, the expression for s, as given by (15), reduces to

$$s = \frac{1 \pm \Delta}{\sqrt{1 \pm k}} \quad .. \quad .. \quad (17)$$

Equation (15) also suggests that the upper and lower frequencies become farther apart as k increases in magnitude.

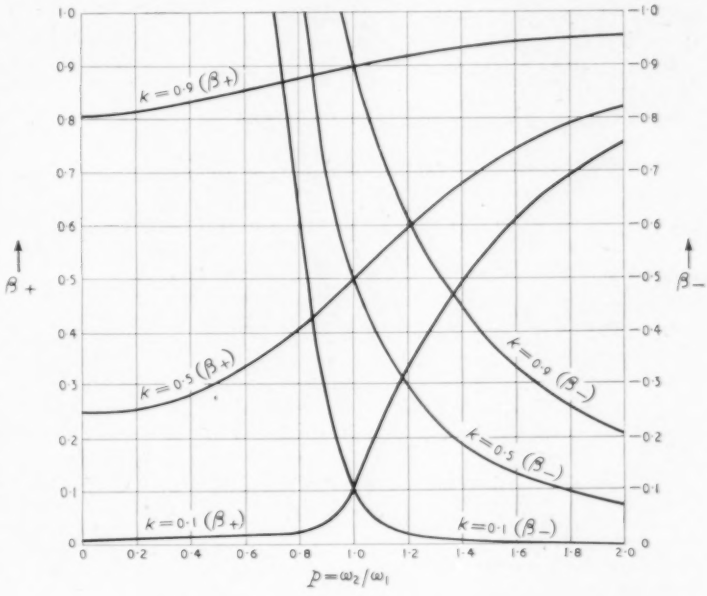

Fig. 3. *Variations of positive and negative values of β as a function of the ratio ω_2/ω_1 for three values of k.*

4. Effects of Resonant Circuit Coupling on Input Conductance

From (11) the total input admittance as seen by the generator will be

$$Y = \frac{1}{R_1(1+\alpha) + j\omega_0 L_1(1-\beta)} + j\omega_0 C_1 \quad (18)$$

and provided $\omega_0 L_1 (1 - \beta) \gg R_1 (1 + \alpha)$, or:

$$s\,\frac{1-\beta}{1+\alpha} \gg \frac{1}{Q_1} \quad .. \quad .. \quad (19)$$

(18) becomes

$$Y = \frac{R_1(1+\alpha)}{\omega_0^2 L_1^2 (1-\beta)^2} + \frac{1}{j\omega_0 L_1(1-\beta)} + j\omega_0 C_1$$

$$= \frac{B_1}{Q_1}\left[\frac{1+\alpha}{s^2(1-\beta)^2} + j\frac{Q_1}{s}\left[(s^2-1) - \frac{\beta}{1-\beta}\right]\right] \quad .. \quad .. \quad .. \quad (20)$$

where B_1 is the magnitude of either primary susceptance at resonance; hence, setting the input susceptance equal to zero yields equation (13).

Since the input conductance at resonance is

$$G_0 = \frac{B_1}{Q_1}\cdot\frac{1+\alpha}{1-\beta} = G_1 \cdot \frac{1+\alpha}{1-\beta} = G_1\phi \quad .. \quad (21)$$

where $G_1 = C_1 R_1/L_1$ is the conductance of the isolated primary circuit at resonance, it may be stated that inductively coupling a secondary circuit to the primary, effectively changes the ratio R_1 and L_1 by a factor ϕ, where

$$\phi = \frac{1+\alpha}{1-\beta} = (1+\alpha)\,s^2 \quad .. \quad .. \quad (22)$$

Furthermore, in view of provision (19), assuming a high-Q circuit, it follows that the resistive, dynamic impedance of a parallel resonant circuit at resonance, $Z_d = L_1/R_1 C_1$, will change by the factor ϕ^{-1} when a secondary tuned circuit is inductively coupled to it; i.e.,

$$Z_d = \frac{L_1}{R_1 C_1}\cdot\left(\frac{1-\beta}{1+\alpha}\right) \quad .. \quad .. \quad .. \quad (23)$$

In order to arrive at the often instructive condition under which the conductance G_0 will have the same value at both upper and lower values of ω_0, let components involving the two possible values of s be denoted by the numerical subscripts 1 and 2, then,

$$\phi_1 = \phi_2 \quad \text{or} \quad \frac{1+\alpha_1}{1-\beta_1} = \frac{1+\alpha_2}{1-\beta_2}$$

which leads to the following required condition:

$$\not{p}^2 + k^2 \frac{Q_2}{Q_1} \not{p} - (1 - k^2) = 0 \quad .. \quad .. \quad (24)$$

solving for k would indicate immediately that this condition exists only for values of $\not{p} < 1$.

Returning to the definition of ϕ as given by (22), let

L_e = equivalent total inductance = $L_1(1 - \beta)$

R_e = equivalent total resistance = $R_1(1 + \alpha)$,

then,

$$\phi = \frac{1+\alpha}{1-\beta} = \frac{R_e/R_1}{L_e/L_1} = \frac{L_1/R_1}{L_e/R_e} = s\frac{Q_1}{Q_0} \quad .. \quad (25)$$

where $Q_1 = Q$ of isolated primary
and $Q_0 = Q$ of coupled primary at resonance.

In the particular case where $\not{p} = 1$, it follows that

$$\not{p} = \pm k, \; \alpha = Q_1/Q_2, \; s = (1 \pm k)^{-\frac{1}{2}},$$

$$G_0 = \frac{B_1}{Q_1}\phi = B_1\left(\frac{1}{Q_1} + \frac{1}{Q_2}\right)\frac{1}{1 \pm k}$$

and $$\frac{1}{Q_0} = \frac{\phi}{sQ_1} = \left(\frac{1}{Q_1} + \frac{1}{Q_2}\right)\frac{1}{\sqrt{1 \pm k}}.$$

5. Frequency Deviation Rate

Expression (15) is the solution of the quadratic equation

$$(1 - k^2)\, s^4 - (1 + \not{p}^2)\, s^2 + \not{p}^2 = 0 \quad (26)$$

which enables one to determine a relationship between small variations of ω_2 and their corresponding effect upon ω_0. It may be shown that

$$\frac{\delta\omega_0}{\delta\omega_2} = \frac{\not{p}s}{\not{p}^2 + k^2/\beta^2} \quad .. \quad .. \quad .. \quad (27)$$

or, in terms of a fractional ratio,

$$\frac{\delta\omega_0/\omega_0}{\delta\omega_2/\omega_2} = \frac{\not{p}^2}{\not{p}^2 + k^2/\beta^2} \quad .. \quad .. \quad .. \quad (28)$$

6. Power Dissipation Ratio

Let P_1 and P_2 denote, respectively, power dissipated in the primary and secondary circuits, then,

$$\frac{P_1}{P_2} = \frac{R_1}{R_2}\left(\frac{I_1}{I_2}\right)^2 \quad .. \quad .. \quad .. \quad .. \quad (29)$$

Applying Kirchhoff's loop equation and allowing for practical values of Q_1 and Q_2 it follows that

$$\frac{P_1}{P_2} = -\frac{\not{p}}{s^2 k^2}\frac{1}{Q_1 Q_2} - j\frac{1}{Q_1}\frac{2}{s\beta} + \frac{k^2 Q_2}{\not{p} Q_1 \beta^2}$$

$$\approx \frac{k^2 Q_2}{\not{p} Q_1}\frac{1}{\beta^2} = \frac{1}{\alpha} \quad .. \quad .. \quad .. \quad (30)$$

Here, if $P_1 = P_2$, α should equal 1; this implies that effective coupled resistance equals primary resistance, which therefore, fulfils the known condition for maximum transfer of power.

7. Stored Energy Ratio

Let W_1 and W_2 represent the energy stored in the primary and secondary circuits, respectively, then,

$$\frac{W_1}{W_2} = \frac{Q_1}{Q_2}\frac{P_1/\omega_1}{P_2/\omega_2} \quad .. \quad .. \quad .. \quad .. \quad (31)$$

and, by substituting (30),

$$\frac{W_1}{W_2} = \left(\frac{k}{\beta}\right)^2 \quad .. \quad .. \quad .. \quad .. \quad (32)$$

Should $W_1 = W_2$, it follows that $\not{p}$ must equal 1, or $\omega_1 = \omega_2$.

Use of (32) may now be made to express the ratio of energy stored in the secondary circuit to the total energy stored in the system; i.e.,

$$\frac{W_2}{W_1 + W_2} = \frac{1}{1 + W_1/W_2} = \frac{1}{1 + (k/\beta)^2} \quad (33)$$

Acknowledgment

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IRREGULAR TRANSMISSION LINES

Extension of Kennelly's Method of Successive Reflections

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SUMMARY.—The Kennelly method of successive reflections is developed to provide general expressions for the voltage and current in an irregular line. The irregularities may have any magnitude, and the labour of summing the reflections is reduced by considering the waves in groups. The expressions are given in terms of the reflection coefficients of the local deviations of the characteristic impedance from a defined reference impedance, and they are later simplified for the common practical conditions in which the impedance deviations are small, e.g., the coaxial cable used for wideband telephony and television transmission. The latter formulae are given in series form for the case where the impedance distribution consists of abrupt steps, and as integrals where the distribution is continuous. Finally, the effects of line irregularities on the open- and short-circuit impedances are briefly considered.

LIST OF PRINCIPAL SYMBOLS

The M.K.S. system is used

$j = \sqrt{-1}$
f = frequency in cycles per second
$\omega = 2\pi f$
R = resistance of uniform line per metre
L = inductance of uniform line per metre
C = capacitance of uniform line per metre
G = leakance of uniform line per metre
V = potential difference
I = current
E = e.m.f. of generator
P = propagation coefficient
γ = propagation coefficient per metre
α = attenuation coefficient, nepers per metre
β = phase-change coefficient, radians per metre
Z = characteristic impedance
Z_m = mean characteristic impedance
Z_r = reference impedance
l = length of line
x = distance of point X from sending end O
p = reflection coefficient
m = transmission coefficient
q = transmitted echo coefficient
M = number of irregular sections in line
σ = deviation of local characteristic impedance from Z_r
S = deviation of local characteristic impedance Z_m

Subscripts

AB pertaining to line AB
A or B pertaining to end A or B of line AB
X pertaining to point X
G pertaining to generator
T pertaining to termination
r pertaining to reference impedance

1. Introduction

PROFESSOR A. E. Kennelly presented his method of obtaining the voltages and currents in a transmission line by means of successive reflections in 1907;[1] the paper was limited to the smooth line with various terminal conditions, and since then the method has been extensively taught as an attractive adjunct to more formal treatments. The principle has been extended to irregular lines (e.g., by Aguillon[6] and Schelkunoff[8]) who obtained their results in terms of the reflections between successive portions of the line; such expressions, however, are somewhat cumbersome and difficult to interpret in practical cases. Others (e.g., Didlaukis and Kaden,[3] Merz and Pfleger[4]) have used the device of decomposing the reflection at any one point into two components, each relative to the mean characteristic impedance, but they deal only with small irregularities.

In this paper the reflections are considered relative to a common reference impedance, and the general case is first established in which the irregularities may have any magnitude; this requires the clear definition of such terms as 'reference impedance' and 'principal wave'. Subsequent sections deal with the nominally homogeneous line in which the irregularities are relatively small, and the nature of the resulting approximations is made apparent.

The line with small irregularities has been the subject of much investigation owing to its importance for coaxial cables used for the transmission of wideband telephony and television. The attractiveness of the method of successive reflections lies in the simple mathematical processes used, and in the close analogy to the actual occurrence of the echoes.

2. Voltage and Current Distribution in the Uniform Line

Before dealing with the irregular line, it is desirable to consider briefly the properties of the uniform line. If we have an infinitely long line with uniformly distributed 'primary parameters', viz., resistance, inductance, capacitance and leakance, all per unit length, and apply at the near end a sinusoidal voltage of frequency f and r.m.s. value V_0, the steady-state voltage and current at a point x metres from the sending end are given by

$$\left.\begin{aligned} V_x &= V_0 \exp(-\gamma x) \\ I_x &= \frac{V_0}{Z} \exp(-\gamma x) \end{aligned}\right\} \qquad \ldots \quad (2{\cdot}1)$$

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MS accepted by the Editor, May 1953.

where

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (2{\cdot}2)$$

$$Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad .. \quad .. \quad .. \quad (2.3)$$

The same distribution of voltage and current occurs if the line is of finite length and terminated at its far end by an impedance equal to its characteristic impedance, Z.

3. The General Irregular Line

3.1. *The Reference Line*

In the general case, irregularities in a line may occur as variations from point to point in any of the four primary parameters, R, L, G, C; correspondingly, there will be variations along the line in the secondary parameters, characteristic impedance and propagation coefficient; i.e., both Z and γ are variables whose value may be designated as Z_x and γ_x respectively at a point X, distant x metres from the sending end. It is only the variation of the local characteristic impedance Z_x along the line which gives rise to reflection effects, and to evaluate them the real line is compared with a hypothetical 'reference line' which retains the same distribution of local propagation coefficient γ_x, but is reflectionless in that it has a uniform characteristic impedance Z_r throughout its length. The value of the 'reference impedance' is arbitrary: for a nominally homogeneous line it is chosen to be close to the average characteristic impedance of the irregular line (defined later), and sometimes it is made equal to this value. The reference line is terminated by an impedance Z_r, and the internal impedance of the generator also has this value.

If such a terminated reference line is fed at the near end O by a generator of impedance Z_r and e.m.f. E, the voltage and current at X are

$$\left.\begin{aligned} V_x &= \tfrac{1}{2}E \exp(-P_{ox}) \\ I_x &= \tfrac{1}{2}E \exp(-P_{ox})/Z_r \end{aligned}\right\} \quad .. \quad .. \quad (3.1)$$

where

$$P_{ox} = \int_0^x \gamma_x dx \quad .. \quad .. \quad .. \quad .. \quad (3.2)$$

3.2. *Method of Successive Reflections*

Kennelly[1] showed that the steady-state complex waves of voltage and current existing when a sine-wave e.m.f. is being sent into a line can be resolved into a series of simple waves, and that the series can be formulated by considering the transmission of fictitious waves of voltage and current which are assumed to reach their steady-state values without any build-up period.

When a wave travelling along a line reaches a discontinuity in impedance, a part of the wave is reflected back and a new wave passes on. Another way of regarding this phenomenon is to consider that the passage of the wave across the boundary between two lines of different characteristic impedances leaves it unchanged, but a pair of equal subsidiary waves is thereby generated, one of which travels on with the originating wave, while the other is reflected back in the reverse direction.

The method of successive reflections consists of introducing hypothetical sine-wave voltage and current waves (the principal waves) at the sending end and following their courses along the line; at every discontinuity they each generate a pair of subsidiary waves, and every subsidiary wave when it in turn crosses a discontinuity generates a further pair of subsidiary waves, the process being continued ad infinitum. We trace the courses of all these waves, and obtain the steady-state voltage and current at any point by adding the separate values of all the waves at that point. The values concerned are vectors, the modulus being the r.m.s. value of the wave and the angle the phase relative to a convenient datum, e.g., the generator e.m.f.; for brevity we shall refer to the magnitude, it being understood that both modulus and angle are thereby implied. In general the process of tracing the waves and adding their magnitudes is a tedious one, but by dealing with the waves in groups the labour is considerably reduced.

The principal waves of voltage and current in an irregular line are defined as the steady-state waves which would flow in the reference line terminated by the reference impedance, when fed by a sine-wave generator having the given e.m.f. and of internal impedance equal to the reference impedance.

It is convenient to distinguish the various waves in the following manner:—plus and minus signs indicate respectively progression in the direction from source to termination and vice versa; a letter indicates the point at which the wave is generated and a number the order of the wave generated at that point, e.g., $+$O1 designates the principal wave, which is the first generated at the origin, and $+$A2 refers to the second forward-travelling wave generated at A.

3.3. *Reflection and Transmission Coefficients*

If V, I, are the magnitudes of the originating voltage and current waves respectively at the boundary between two semi-infinite lines (Fig. 1), then the magnitudes of the subsidiary voltage and current waves V_s, I_s, are given by

$$\left.\begin{aligned} V_s &= pV \\ I_s &= -pI \end{aligned}\right\} \quad .. \quad .. \quad .. \quad .. \quad (3.3)$$

where

$$p = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad .. \quad .. \quad .. \quad .. \quad (3.4)$$

Z_2, Z_1, being the respective characteristic impedances of the lines entered and left by the originating wave.

The reflection coefficient of a single discontinuity is defined as the vector ratio of the magnitudes of the reflected and incident waves; similarly, the transmission coefficient is the vector ratio of the magnitudes of the transmitted and incident waves. Referring to Fig. 1, the voltage reflection coefficient is

$$p = \frac{V_s}{V} = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad .. \quad .. \quad .. \quad (3.5)$$

The current reflection coefficient is

$$p' = \frac{I_s}{I} = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -p \quad .. \quad .. \quad (3.6)$$

The voltage transmission coefficient is

$$m = \frac{V + V_s}{V} = \frac{2Z_2}{Z_1 + Z_2} = 1 + p \quad .. \quad (3.7)$$

The current transmission coefficient is

$$m' = \frac{I + I_s}{I} = \frac{2Z_1}{Z_1 + Z_2} = 1 - p \quad .. \quad (3.8)$$

It will be seen that, for a single discontinuity, the voltage and current reflection coefficients are equal and of opposite sign, whereas the voltage and current transmission coefficients are unequal.

The concept may be extended to cover reflection from and transmission through the whole or any portion of a line. In this case, the reflection coefficient p is defined as the vector ratio of the sum of the reflected waves at the near end of the portion to the principal wave at that point; similarly the transmission coefficient m is the vector ratio of the sum of all the transmitted waves both principal and subsidiary at the far end of the portion to the principal wave at that point. The transmitted echo coefficient q, corresponding to the German term 'Mitfluszfaktor' and the French term 'coefficient de trainage', is the vector ratio of the sum of the subsidiary transmitted waves at the far end of the portion to the principal wave at that point; it is thus equal to $m - 1$.

The coefficients will vary according to the impedances of the generator and termination. When the termination and generator have the same impedance, they are said to be matched, and unless the contrary is stated, the coefficients used below refer to matched terminal impedances equal to the reference impedance. Further, the coefficients refer to the voltage waves unless otherwise stated.

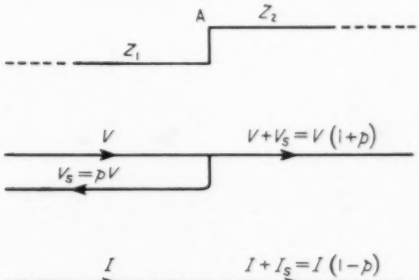


Fig. 1. *Transmitted and reflected waves at a single impedance discontinuity.*

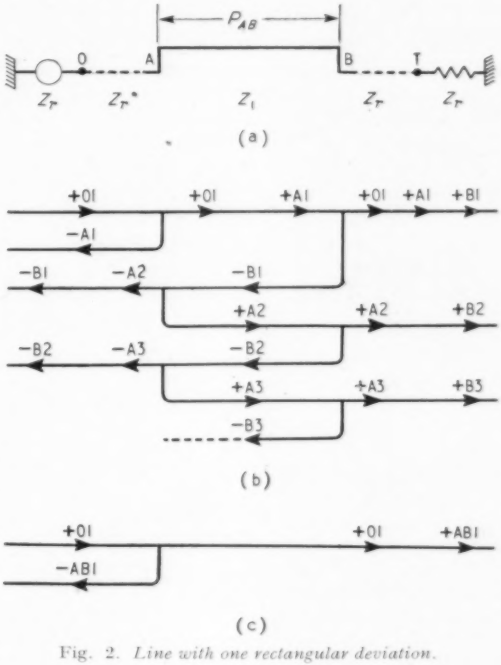


Fig. 2. *Line with one rectangular deviation.*

3.4. *Line with One Rectangular Deviation*

Consider a line of characteristic impedance Z_r, with generator at O and termination at T, both of impedance Z_r [Fig. 2(a)]; inserted in the line between points A and B is a portion of characteristic impedance Z_1 and propagation coefficient P_{AB}, termed a 'rectangular' deviation, from the shape of the diagram. The reflection coefficients relating to the discontinuities at A and B have opposite signs, since, for example, forward-travelling waves at A pass from Z_r to Z_1, whereas at B they pass from Z_1 to Z_r. Forward-travelling waves in portion BT are absorbed at T without reflection, and similarly backward-travelling waves in AO are absorbed at O without reflection.

Considering the voltage along the line [Fig. 2(b)], let the magnitude of the principal wave + O1 at A be V; then its magnitude at B is

$V \exp(-P_{AB})$. At A, two subsidiary waves $\pm$ A1 of magnitude Vp_A are generated, where

$$p_A = \frac{Z_1 - Z_T}{Z_1 + Z_T} \quad .. \quad .. \quad .. \quad .. \quad (3.9)$$

Wave $-$ A1 travels back to the source, is absorbed there, and does not generate any further subsidiary waves; $+$ A1 travels forward with the principal wave, making the sum of the two waves at B equal to $V(1 + p_A) \exp(-P_{AB})$.

At B, a pair of subsidiary waves $\pm$ B1 are generated, each of magnitude $-Vp_A(1 + p_A) \exp(-P_{AB})$. The sum of $+$ O1, $+$ A1 and $+$ B1 leaving B towards T is thus

$$V(1 - p^2_A) \exp(-P_{AB}).$$

Let us now trace $-$ B1 travelling from B to A. When it reaches A its magnitude is $-Vp_A(1 + p_A) \exp(-2P_{AB})$; it continues on towards O and in crossing the junction at A, a pair of new waves $\pm$ A2 is generated of magnitude

$$Vp^2_A(1 + p_A) \exp(-2P_{AB}).$$

Of these, $-$ A2 travels on to the source and $+$ A2 goes forward to B. The sum of the waves $-$ A1, $-$ B1, $-$ A2 returning towards O at this stage is

$$Vp_A\{1 - (1 - p^2_A) \exp(-2P_{AB})\}.$$

Wave $+$ A2 travelling forward has a magnitude $Vp^2_A(1 + p_A) \exp(-3P_{AB})$ at B; it goes on towards T, and in crossing the junction at B it generates $\pm$B2 of magnitude $-Vp^3_A(1 + p_A) \exp(-3P_{AB})$, so that the total going forward to T is now $V[(1 - p^2_A) \exp(-P_{AB}) + (p^2_A - p^4_A) \exp(-3P_{AB})]$. Wave $-$ B2 returns to A, and the process of reflection back and forth between A and B is continued ad infinitum. The magnitudes of the principal and subsidiary waves on the source side of junction A and on the termination side of junction B are given in Table 1.

The total voltage at A is

$$V_A = V[1 + \{1 - \exp(-2P_{AB})\}\{p_A + p^3_A \exp(-2P_{AB}) + p^5_A \exp(-4P_{AB}) + - - - \}] \quad .. \quad .. \quad .. \quad .. \quad (3.10)$$

$$= \frac{V(1 + p_A)\{1 - p_A \exp(-2P_{AB})\}}{1 - p^2_A \exp(-2P_{AB})} \quad .. \quad (3.11)$$

The total voltage at B is

$$V_B = V \exp(-P_{AB}) \{1 - p^2_A + p^2_A \exp(-2P_{AB}) - p^4_A \exp(-2P_{AB}) + p^4_A \exp(-4P_{AB}) - -\} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3.12)$$

$$= \frac{V(1 - p^2_A) \exp(-P_{AB})}{1 - p^2_A \exp(-2P_{AB})} \quad .. \quad .. \quad (3.13)$$

The voltage at A, as shown by equation (3.10), consists of the principal wave and a series containing only odd powers of p_A. The voltage at B consists of the principal wave and a series of even powers of p_A.

The current in the line is obtained from the voltage equations by reversing the sign of the odd powers of p_A, the sign of the even powers remaining unchanged. If I is the magnitude of the principal current wave at A, the current at A, I_A, is given by

$$I_A = \frac{I(1 - p_A)\{1 + p_A \exp(-2P_{AB})\}}{1 - p^2_A \exp(-2P_{AB})} \quad (3.14)$$

The current at B is

$$I_B = \frac{I(1 - p^2_A) \exp(-P_{AB})}{1 - p^2_A \exp(-2P_{AB})} \quad .. \quad (3.15)$$

The impedance at A looking towards B is from (3.11) and (3.14)

$$Z_{AB} = \frac{Z_T(1 + p_A)\{1 - p_A \exp(-2P_{AB})\}}{(1 - p_A)\{1 + p_A \exp(-2P_{AB})\}} \quad .. \quad .. \quad .. \quad (3.16a)$$

$$= Z_1 \frac{1 - p_A \exp(-2P_{AB})}{1 + p_A \exp(-2P_{AB})} \quad .. \quad (3.16b)$$

$$= Z_1 \frac{Z_T \cosh P_{AB} + Z_1 \sinh P_{AB}}{Z_T \sinh P_{AB} + Z_1 \cosh P_{AB}} \quad .. \quad (3.17)$$

TABLE 1

Waves in Line with One Rectangular Deviation

Wave	Magnitude in OA at A	Magnitude in BT at B
$+$ O1	V	$V \exp(-P_{AB})$
$-$ A1	Vp_A	—
$+$ A1	—	$Vp_A \exp(-P_{AB})$
$+$ B1	—	$-Vp_A(1 + p_A) \exp(-P_{AB})$
$-$ B1	$-Vp_A(1 + p_A) \exp(-2P_{AB})$	—
$-$ A2	$Vp^2_A(1 + p_A) \exp(-2P_{AB})$	—
$+$ A2	—	$Vp^2_A(1 + p_A) \exp(-3P_{AB})$
$+$ B2	—	$-Vp^3_A(1 + p_A) \exp(-3P_{AB})$
$-$ B2	$-Vp^3_A(1 + p_A) \exp(-4P_{AB})$	—
$-$ A3	$Vp^4_A(1 + p_A) \exp(-4P_{AB})$	—
$+$ A3	—	$Vp^4_A(1 + p_A) \exp(-5P_{AB})$
$+$ B3	—	$-Vp^5_A(1 + p_A) \exp(-5P_{AB})$
$-$ B3	$-Vp^5_A(1 + p_A) \exp(-6P_{AB})$	—
etc.	- - -	- - -

The voltage reflection coefficient of portion AB at A is, from equation (3.11)

$$p_{AB} = \frac{p_A\{1 - \exp(-2P_{AB})\}}{1 - p^2_A \exp(-2P_{AB})} \quad .. \quad (3.18)$$

and by symmetry, the voltage reflection coefficient at B, p_{BA}, is equal to p_{AB}. From equation (3.14) it is seen that the current reflection coefficient is equal and opposite to the voltage reflection coefficient.

The voltage transmission coefficient is from (3.13)

$$m_{AB} = \frac{1 - p^2_A}{1 - p^2_A \exp(-2P_{AB})} = 1 - p_A p_{AB} \quad .. \quad .. \quad .. \quad (3.19)$$

Equation (3.15) shows that the current transmission coefficient is equal to the voltage transmission coefficient.

We may form the subordinate waves caused by the passage of the principal wave through the portion AB into two groups. The first group consisting of all waves travelling back from A is designated $-$ AB1; the second, containing all subordinate waves travelling forward from the remote end B is designated $+$ AB1, and we may draw Fig. 2(c) to represent the same series of waves as is shown in Fig. 2(b).

3.5. *Line with Two Rectangular Deviations*

Consider a line OT of impedance Z_T (Fig. 3) in which are inserted a smooth portion AB of impedance Z_1 and propagation coefficient P_{AB} and a second smooth portion CD of impedance Z_2 and propagation coefficient P_{CD}. From Fig. 3 we can construct Table 2, which gives the magnitudes of the various wave-groups at A, B, C and D. The total voltage at A is

$$V_A = V[1 + p_{AB} + p_{CD}m^2_{AB}\exp(-2P_{AB} - 2P_{BC}) + p_{BA}p^2_{CD}m^2_{AB}\exp(-2P_{AB} - 4P_{BC}) + \cdots] \quad .. \quad .. \quad .. \quad (3.20)$$

$$= V\left[1 + p_{AB} + \frac{p_{CD}m^2_{AB}\exp(-2P_{AC})}{1 - p_{BA}p_{CD}\exp(-2P_{BC})}\right] \quad .. \quad .. \quad .. \quad (3.21)$$

The total voltage at D is

$$V_D = V\{m_{AB}m_{CD} + p_{BA}p_{CD}m_{AB}m_{CD}\exp(-2P_{BC}) + p^2_{BA}p^2_{CD}m_{AB}m_{CD}\exp(-4P_{BC}) + \cdots\}\exp(-P_{AD}) \quad .. \quad .. \quad .. \quad (3.22)$$

$$= \frac{Vm_{AB}m_{CD}\exp(-P_{AD})}{1 - p_{BA}p_{CD}\exp(-2P_{BC})} \quad .. \quad .. \quad (3.23)$$

The voltage reflection coefficient for the portion AD is, from (3.21)

TABLE 2

Line with Two Rectangular Deviations

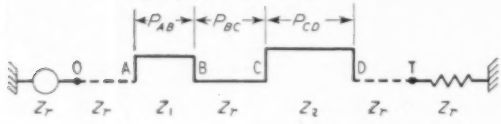
Magnitude of Voltage Waves relative to the Magnitude of the Principal Wave at the Same Point

Wave group	Magnitude at A	Magnitude at B	Magnitude at C	Magnitude at D
+ O1	1	1	1	1
− AB1	p_{AB}	—	—	—
+ AB1	—	$m_{AB} - 1$	$m_{AB} - 1$	$m_{AB} - 1$
+ CD1	—	—	—	$m_{AB}(m_{CD} - 1)$
− CD1	$p_{CD}m_{AB}\exp(-2P_{AB} - 2P_{BC})$	$p_{CD}m_{AB}\exp(-2P_{BC})$	$p_{CD}m_{AB}$	—
− AB2	$p_{CD}m_{AB}(m_{AB} - 1)\exp(-2P_{AB} - 2P_{BC})$	—	—	—
+ AB2	.	$p_{BA}p_{CD}m_{AB}\exp(-2P_{BC})$	$p_{BA}p_{CD}m_{AB}\exp(-2P_{BC})$	$p_{BA}p_{CD}m_{AB}\exp(-2P_{BC})$
+ CD2	—	—	—	$p_{BA}p_{CD}m_{AB}(m_{CD} - 1)\exp(-2P_{BC})$
− CD2	$p_{BA}p^2_{CD}m_{AB}\exp(-2P_{AB} - 4P_{BC})$	$p_{BA}p^2_{CD}m_{AB}\exp(-4P_{BC})$	$p_{BA}p^2_{CD}m_{AB}\exp(-2P_{BC})$	—
− AB3	$p_{BA}p^2_{CD}m_{AB}(m_{AB} - 1)\exp(-2P_{AB} - 4P_{BC})$	—	—	—
+ AB3	—	$p^2_{BA}p^2_{CD}m_{AB}\exp(-4P_{BC})$	$p^2_{BA}p^2_{CD}m_{AB}\exp(-4P_{BC})$	$p^2_{BA}p^2_{CD}m_{AB}\exp(-4P_{BC})$
+ CD3	—	—	—	$p^2_{BA}p^2_{CD}m_{AB}(m_{CD} - 1)\exp(-4P_{BC})$
etc.	. . .	. . .	. . .	. . .

$$p_{AD} = p_{AB} + \frac{p_{CD}m^2_{AB}\exp(-2P_{AC})}{1 - p_{BA}p_{CD}\exp(-2P_{BC})} \quad (3.24)$$

The voltage transmission coefficient for AD is, from (3.23)

$$m_{AD} = \frac{m_{AB}m_{CD}}{1 - p_{BA}p_{CD}\exp(-2P_{BC})} \quad .. \quad (3.25)$$



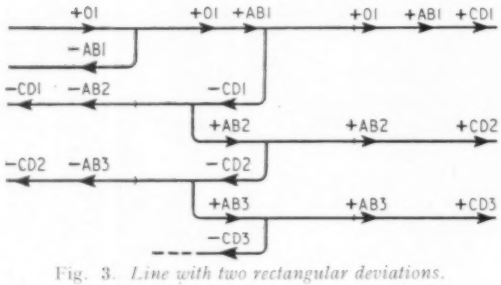


Fig. 3. *Line with two rectangular deviations.*

If generator and termination are interchanged in Fig. 3, we obtain the reflection and transmission coefficients for transmission through the line from D to A. From (3.24) and (3.25) we obtain from considerations of symmetry

$$p_{DA} = p_{DC} + \frac{p_{BA}m^2_{CD}\exp(-2P_{BD})}{1 - p_{BA}p_{CD}\exp(-2P_{BC})} \quad (3.26)$$

$$m_{DA} = \frac{m_{AB}m_{CD}}{1 - p_{BA}p_{CD}\exp(-2P_{BC})} = m_{AD} \quad (3.27)$$

If the two portions are adjacent, $P_{BC} = 0$. Then for two adjacent rectangular deviations,

$$p_{AC} = p_{AB} + \frac{p_{BC}m^2_{AB}\exp(-2P_{AB})}{1 - p_{BA}p_{BC}} \quad .. \quad (3.28)$$

$$p_{CA} = p_{CB} + \frac{p_{BA}m^2_{BC}\exp(-2P_{BC})}{1 - p_{BA}p_{BC}} \quad .. \quad (3.29)$$

$$m_{AC} = m_{CA} = \frac{m_{AB}m_{BC}}{1 - p_{BA}p_{BC}} \quad .. \quad .. \quad (3.30)$$

By following a similar procedure for the current waves, we obtain equations corresponding to (3.24), (3.25), (3.26) and (3.27); we see that for two rectangular deviations the voltage and current reflection coefficients at the same end are equal and opposite to each other, while the voltage and current transmission coefficients are equal to each other and are the same for either direction of transmission.

3.6. *Line with M Adjacent Rectangular Deviations*

If the line contains three rectangular deviations adjacent to each other, AB, BC, CD, we treat it as two portions AC, CD, and by following the process used for two rectangular deviations we arrive at the equations

$$\begin{aligned} p_{AD} &= p_{AC} + \frac{m^2_{AC}p_{CD}\exp(-2P_{AC})}{1 - p_{CA}p_{CD}} \\ &= p_{AB} + \frac{m^2_{AB}p_{BC}\exp(-2P_{AB})}{1 - p_{BA}p_{BC}} \\ &\quad + \frac{m^2_{AC}p_{CD}\exp(-2P_{AC})}{1 - p_{CA}p_{CD}} \quad .. \quad (3.31) \end{aligned}$$

$$m_{AD} = \frac{m_{AC}m_{CD}}{1 - p_{CA}p_{CD}} = \frac{m_{AB}m_{BC}m_{CD}}{(1 - p_{BA}p_{BC})(1 - p_{CA}p_{CD})} \quad .. \quad .. \quad .. \quad (3.32)$$

When the line contains M adjacent rectangular deviations AB, BC . . . MN, we obtain by continuing the process

$$\begin{aligned} p_{AN} &= p_{AB} + \frac{m^2_{AB}p_{BC}\exp(-2P_{AB})}{1 - p_{BA}p_{BC}} \\ &\quad + \frac{m^2_{AC}p_{CD}\exp(-2P_{AC})}{1 - p_{CA}p_{CD}} \\ &\quad + \text{- - - -} + \frac{m^2_{AM}p_{MN}\exp(-2P_{AM})}{1 - p_{MA}p_{MN}} \\ &\quad .. \quad .. \quad .. \quad (3.33) \end{aligned}$$

$$m_{AN} = m_{NA} = \frac{m_{AB}m_{BC} \ldots\ldots m_{MN}}{(1 - p_{BA}p_{BN})(1 - p_{CB}p_{CN}) \ldots\ldots (1 - p_{LM}p_{MN})} \quad .. \quad (3.34)$$

$$= \frac{m_{AB}m_{BC} \ldots\ldots m_{MN}}{(1 - p_{BA}p_{BC})(1 - p_{CA}p_{CD}) \ldots\ldots (1 - p_{MA}p_{MN})} \quad .. \quad (3.35)$$

3.7. *Line with Deviations of any Form*

A deviation AB of any form may be regarded as one composed of a large number of adjacent rectangular deviations of very short length. The forward-travelling subsidiary waves may be grouped together to form a single wave-group + AB1, and the backward-travelling subsidiaries may be grouped to form a second wave-group − AB1, the whole process being represented by Fig. 2(c). Thus Sections (3.5) and (3.6), with the contained equations (3.20) to (3.35) inclusive and Table 2, apply mutatis mutandis to lines with deviations of any form.

The following relations pertain to a line AB containing deviations of any form, with matched terminal conditions:—

(*a*) The voltage and current reflection coefficients at the same end are equal and opposite in sign to each other.

(*b*) It follows from (*a*) that if p_{AB} is the voltage reflection coefficient, Z_{AB} the impedance looking

into the line at A and Z_T the impedance of the generator and termination

$$V_A = V(1 + p_{AB})$$
$$I_A = I(1 - p_{AB})$$

whence $$Z_{AB} = \frac{Z_T(1 + p_{AB})}{1 - p_{AB}} \quad .. \quad .. \quad (3.36)$$

$$p_{AB} = \frac{Z_{AB} - Z_T}{Z_{AB} + Z_T} \quad .. \quad .. \quad .. \quad (3.37)$$

(*c*) The reflection coefficients at end B in general bear no relation to the reflection coefficients at end A.

(*d*) The voltage and current transmission coefficients are equal to each other.

(*e*) The voltage and current transmission coefficients are the same for either direction of transmission; this may be deduced directly from the Reciprocity Theorem.

3.8. *Irregular Line with General Terminal Conditions*

If an irregular line is terminated with an impedance Z_T other than the reference impedance Z_r, the voltage reflection coefficient p_{ABT} may be obtained from equation (3.28) in its general application by substituting p_T for p_{BC}, where

$$Z_{ABT} = \frac{Z_r(1 + p_{ABT})}{1 - p_{ABT}} = Z_r\left[\frac{(1 + p_{AB})(1 - p_{BA}p_T) + p_T m^2_{AB} \exp(-2P_{AB})}{(1 - p_{AB})(1 - p_{BA}p_T) - p_T m^2_{AB} \exp(-2P_{AB})}\right] \quad .. \quad (3.47)$$

$$p_T = \frac{Z_T - Z_r}{Z_T + Z_r} \quad .. \quad .. \quad .. \quad (3.38)$$

Thus $$p_{ABT} = p_{AB} + \frac{p_T m^2_{AB} \exp(-2P_{AB})}{1 - p_{BA}p_T} \quad (3.39)$$

Similarly, for the opposite direction of transmission

$$p_{BAT} = p_{BA} + \frac{p_T m^2_{AB} \exp(-2P_{AB})}{1 - p_{AB}p_T} \quad (3.40)$$

The voltage transmission coefficient m_{ABT} is obtained from equation (3.30) by substituting $(1 + p_T)$ for m_{BC}. Thus

$$m_{ABT} = \frac{m_{AB}(1 + p_T)}{1 - p_{BA}p_T} \quad .. \quad .. \quad (3.41)$$

$$m_{BAT} = \frac{m_{AB}(1 + p_T)}{1 - p_{AB}p_T} \quad .. \quad .. \quad (3.42)$$

The current reflection coefficient in terms of the voltage coefficients for the component parts is

$$p'_{ABT} = -p_{AB} - \frac{p_T m^2_{AB} \exp(-2P_{AB})}{1 - p_{BA}p_T} \quad .. \quad .. \quad .. \quad .. \quad (3.43)$$

Similarly the current transmission coefficient is

$$m'_{ABT} = \frac{m_{AB}(1 - p_T)}{1 - p_{BA}p_T} \quad .. \quad .. \quad (3.44)$$

The current reflection coefficient is equal and opposite to the voltage reflection coefficient, so that the relationships between impedance and reflection coefficients, equations (3.36) and (3.37), hold good. The voltage and current transmission coefficients, however, are no longer equal to each other, and furthermore, they differ for the two directions of transmission.

The voltage at the sending end is

$$V_A = V(1 + p_{ABT}) = V\left\{1 + p_{AB} + \frac{p_T m^2_{AB} \exp(-2P_{AB})}{1 - p_{BA}p_T}\right\} \quad .. \quad .. \quad .. \quad .. \quad (3.45)$$

where V is the magnitude of the principal voltage wave at the sending end. The current at the sending end is

$$I_A = I\left\{1 - p_{AB} - \frac{p_T m^2_{AB} \exp(-2P_{AB})}{1 - p_{BA}p_T}\right\} \quad .. \quad .. \quad .. \quad .. \quad (3.46)$$

where I is the magnitude of the principal current wave at the sending end. Thus the sending-end impedance is

If the generator impedance is changed and its e.m.f. remains unaltered at E, the principal voltage wave, by definition, is unaffected. If the new value of the generator impedance is Z_G, the voltage at the sending end of the line terminated by Z_T becomes $EZ_{ABT}/(Z_G + Z_{ABT})$. The corresponding value of the principal wave is $\frac{1}{2}E$, the new reflected wave is $\frac{1}{2}E(Z_{ABT} - Z_G)/(Z_{ABT} + Z_G)$, and thus the new reflection coefficient p_{GABT} is

$$p_{GABT} = \frac{Z_{ABT} - Z_G}{Z_{ABT} + Z_G} \quad .. \quad .. \quad .. \quad (3.48)$$

which is in agreement with the accepted definition of reflection coefficient. From equations (3.47) and (3.48) we have

$$p_{GABT} = \frac{(p_{AB} + p_G)(1 - p_{BA}p_T) + p_T m^2_{AB} \exp(-2P_{AB})}{(1 + p_{AB}p_G)(1 - p_{BA}p_T) + p_G p_T m^2_{AB} \exp(-2P_{AB})} \quad .. \quad (3.49)$$

where $p_G = (Z_T - Z_G)/(Z_T + Z_G)$ .. (3.50)

The effect of changing the generator impedance from Z_T to Z_G is to alter the received voltage in the ratio $(Z_{ABT} + Z_T)/(Z_{ABT} + Z_G)$. Since the principal wave is unaltered, the new transmission coefficient m_{GABT} becomes

$$m_{GABT} = \frac{m_{ABT}(Z_{ABT} + Z_T)}{Z_{ABT} + Z_G} = \frac{m_{AB}(1 + p_G)(1 + p_T)}{(1 + p_{AB}p_G)(1 - p_{BA}p_T) + p_G p_T m^2_{AB} \exp(-2P_{AB})} \quad .. \quad (3.51)$$

If $Z_T = Z_G$ (i.e., the terminations are matched at the new value Z_G instead of Z_T) $p_G = -p_T$, and

$$p_{GABG} = \frac{(p_{AB} + p_G)(1 + p_{BA}p_G) - p_G m^2_{AB} \exp(-2P_{AB})}{(1 + p_{AB}p_G)(1 + p_{BA}p_G) - p^2_G m^2_{AB} \exp(-2P_{AB})} \quad .. \quad (3.52)$$

$$m_{GABG} = \frac{m_{AB}(1 - p^2_G)}{(1 + p_{AB}p_G)(1 + p_{BA}p_G) - p^2_G m^2_{AB} \exp(-2P_{AB})} \quad .. \quad (3.53)$$

In the above, the coefficients p_{AB}, p_{BA}, m_{AB} relate to the reference impedance Z_T. Changing the reference impedance will not alter either the actual line voltages or the principal voltage wave, and hence p_{GABG} and m_{GABG} are unaffected by a change in the reference impedance, e.g., from Z_T to Z_G. Thus equations (3.52) and (3.53) give the change in the reflection and transmission coefficients respectively when the reference impedance is changed from Z_T to Z_G.

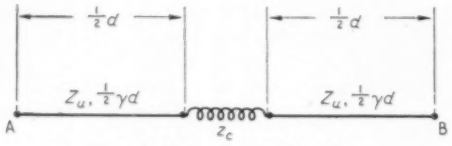


Fig. 4. *Section of loaded cable.*

3.9. *Localized Irregularities*

A localized irregularity is one in which a relatively large impedance deviation has such a small propagation coefficient that it is effectively concentrated at a single point; e.g., a series inductor or a shunt capacitor. The reflection and transmission coefficients can be obtained directly from the definitions and the elementary circuit diagrams, but it is instructive to see how they follow from the general case. The propagation coefficient being very small, the reflection coefficient is from equation (3.18)

$$p_{AB} = \frac{2P_{AB}p_A}{1 - p^2_A(1 - 2P_{AB})} = \frac{2P_{AB}(Z^2_1 - Z^2_T)}{4Z_1Z_T + 2P_{AB}(Z_1 - Z_T)^2} \quad .. \quad (3.54)$$

If the deviation is due to a relatively large series impedance z_c associated with a small shunt admittance y_c (such as a loading coil), $Z^2_1 = z_c/y_c$ is large compared with Z^2_T, and hence

$$p_{AB} = \frac{P_{AB}Z_1}{2Z_T + P_{AB}Z_1}$$

Since $p_{AB} = \sqrt{z_c y_c}$, we have

$$p_{AB} = \frac{z_c}{2Z_T + z_c} \quad .. \quad .. \quad .. \quad (3.55)$$

From equation (3.19), since $p_A \approx 1$,

$$m_{AB} = 1 - \frac{z_c}{2Z_T + z_c} = \frac{2Z_T}{2Z_T + z_c} \quad .. \quad (3.56)$$

On the other hand if the shunt admittance is relatively large and the series impedance small, Z^2_1 can be neglected relative to Z^2_T, and $p_A \approx -1$. Whence

$$p_{AB} = \frac{-P_{AB}Z_T}{2Z_1 + P_{AB}Z_T} = \frac{-y_c Z_T}{2 + y_c Z_T} \quad .. \quad (3.57)$$

$$m_{AB} = 1 - \frac{y_c Z_T}{2 + y_c Z_T} = \frac{2}{2 + y_c Z_T} \quad .. \quad (3.58)$$

It is interesting to use these relations to obtain Campbell's formula for the loaded cable. Referring to Fig. 4, AB represents a loading coil of impedance z_c inserted between two portions of cable, each of length $\frac{1}{2}d$, of characteristic impedance Z_u and propagation coefficient γ per unit length. If we choose the reference impedance to be Z_u, we have

$$p_{AB} = \frac{z_c \exp(-\gamma d)}{z_c + 2Z_u}; \quad m_{AB} = \frac{2Z_u}{z_c + 2Z_u} \quad .. \quad .. \quad .. \quad (3.59)$$

The impedance with end B open-circuited is obtained from equation (3.47), for which $p_T = +1$; whence

$$Z_{op} = Z_u \left[\frac{1 - p^2_{AB} + m^2_{AB} \exp(-2\gamma d)}{(1 - p_{AB})^2 - m^2_{AB} \exp(-2\gamma d)} \right]$$

Similarly with end B short-circuited, $p_T = -1$, and $Z_{sh} = Z_u \left[\frac{(1 + p_{AB})^2 - m^2_{AB} \exp(-2\gamma d)}{1 - p^2_{AB} + m^2_{AB} \exp(-2\gamma d)} \right]$

If P' is the propagation coefficient per unit length of the loaded cable, then

$$\tanh^2 P'd = \frac{Z_{sh}}{Z_{op}} = 1 - \frac{4m^2_{AB} \exp(-2\gamma d)}{\{1 - p^2_{AB} + m^2_{AB} \exp(-2\gamma d)\}^2}$$

Whence

$$\cosh P'd = \frac{1 - p^2_{AB} + m^2_{AB} \exp(-2\gamma d)}{2m_{AB} \exp(-\gamma d)}$$

and substituting from (3.59) we have

$$\cosh P'd = \cosh \gamma d + \frac{z_c}{2Z_u} \sinh \gamma d \quad .. \quad (3.60)$$

which is Campbell's well-known formula.

4. Line with Small Irregularities

4.1. *Simplification when Impedance Deviations are Small*

If the impedance deviations are small relative to the reference impedance, the reflection coefficients p_A, p_B, etc., are small compared with unity. The coefficients for one deviating smooth portion from (3.18) and (3.19) simplify to

$$p_{AB} = p_A \{1 - \exp(-2P_{AB})\} \quad .. \quad (4.1)$$

$$m_{AB} = 1 - p^2{}_A \{1 - \exp(-2P_{AB})\} \quad .. \quad (4.2)$$

Considering M adjacent smooth portions, we find by substituting (4.1) and 4.2) in equation (3.33) that terms of the type $m^2{}_{AB}p_{BC}/(1 - p_{BA}p_{BC})$ reduce to p_{BC}. Thus we have

$$p_{AN} = p_{AB} + p_{BC}\exp(-2P_{AB}) + p_{CD}\exp(-2P_{AC}) + \text{- - - - - +} p_{MN}\exp(-2P_{AM}) \quad .. \quad .. \quad (4.3)$$

Equation (4.3) shows that the reflected wave from each deviating portion is the same as if the line between it and the source were smooth; further, only single reflections and consequently first powers of reflection coefficients are of importance. The corresponding wave-diagram is shown in Fig. 5.

The transmission coefficient for two deviating portions, from (3.25) reduces to

$$\begin{aligned} m_{AD} &= m_{AB}m_{CD}\{1 + p_{BA}p_{CD}\exp(-2P_{BC})\} \\ &= 1 - p^2{}_A\{1 - \exp(-2P_{AB})\} \\ &\quad - p^2{}_C\{1 - \exp(-2P_{CD})\} \\ &\quad + p_A p_C \exp(-2P_{BC})\{1 - \exp(-2P_{AB})\} \\ &\quad \{1 - \exp(-2P_{CD})\} \quad .. \quad .. \quad .. \quad (4.4) \end{aligned}$$

Only second powers of reflection coefficients and double reflections are concerned; the corresponding wave-diagram is shown in Fig. 6.

For M adjacent smooth portions, the transmission coefficient, from (3.34) becomes

$$m_{AN} = m_{AB}m_{BC}m_{CD} \ldots\ldots m_{MN} (1 + p_{BA}p_{BC} + p_{CA}p_{CD} + \text{- - - -} + p_{MA}p_{MN}) \quad (4.5)$$

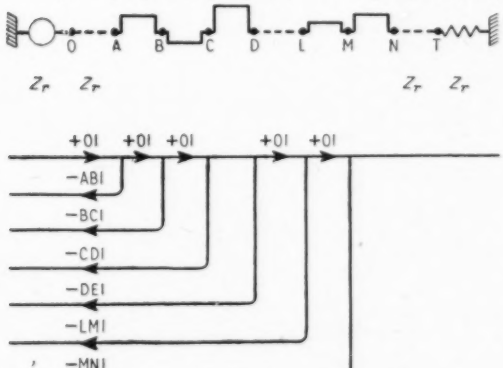


Fig. 5. *Single reflections in line with small rectangular deviations.*

$$\begin{aligned} &= 1 - p^2{}_A\{1 - \exp(-2P_{AB})\} \\ &\quad - p^2{}_B\{1 - \exp(-2P_{BC})\} - \\ &\quad \text{- - - -} - p^2{}_M\{1 - \exp(-2P_{MN})\} \\ &\quad + p_{BC}p_{BA} + p_{CD}\{p_{CB} + p_{BA}\exp(-2P_{BC})\} \\ &\quad + \text{- - - -} + p_{MN}\{p_{ML} + p_{LK}\exp(-2P_{LM}) \\ &\quad + \text{- - - -} + p_{BA}\exp(-2P_{BM})\} \\ &= 1 - \sum_{F=A}^{M} p^2{}_F\{1 - \exp(-2P_{FG})\} + \\ &\qquad \sum_{F=B}^{M} p_{FG}\left\{\sum_{D=B}^{F} p_{DC}\exp(-2P_{DF})\right\} \\ &= 1 - \sum_{F=A}^{M} p^2{}_F\{1 - \exp(-2P_{FG})\} \\ &\quad + \sum_{F=B}^{M} p_F\{1 - \exp(-2P_{FG})\} \\ &\quad \left[\sum_{D=B}^{F} p_C\{1 - \exp(-2P_{CD})\}\exp(-2P_{DF})\right] \quad (4.6) \end{aligned}$$

By similar reasoning to the above, we can see that (4.3) and (4.5) apply also to an arrangement of M adjacent irregular portions.

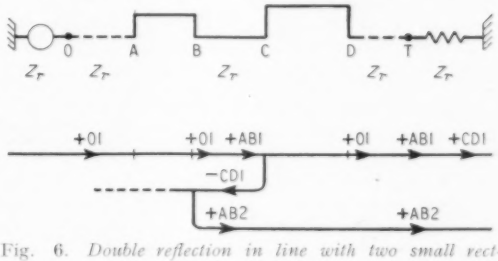


Fig. 6. *Double reflection in line with two small rectangular deviations.*

4.2. *Irregular Line with General Terminal Conditions*

Consider the irregular line AB treated in Section (3.8) in which the generator impedance is changed from Z_r to Z_G and the terminating impedance from Z_r to Z_T; when the line irregularities are small the reflection coefficient p_{GABT} in (3.49) becomes

$$p_{GABT} = \frac{(p_{AB} + p_G)(1 - p_{BA}p_T) + p_T\exp(-2P_{AB})}{1 + p_{AB}p_G - p_{BA}p_T + p_G p_T\exp(-2P_{AB})}$$

If $p_G p_T \exp(-2P_{AB})$ is small compared with 1, which is often the case,

$$p_{GABT} = p_{AB} + p_G + p_T\exp(-2P_{AB}) - p_{AB}p^2{}_G \quad .. \quad .. \quad .. \quad .. \quad (4.7)$$

From (3.51) we obtain for the transmission coefficient

$$m_{GABT} = m_{AB}(1 + p_G)(1 + p_T)\{1 - p_{AB}p_G + p_{BA}p_T - p_G p_T\exp(-2P_{AB})\} \quad .. \quad .. \quad .. \quad (4.8)$$

If q_{AB} denotes the transmitted echo coefficient when generator and termination are equal to Z_r, and q_{GABT} the value when these impedances are respectively Z_G and Z_T, then

$$q_{GABT} = (1 + p_G)(1 + p_T)\{q_{AB} - p_{AB}p_G + p_{BA}p_T - p_G p_T \exp(-2P_{AB})\} \quad .. \quad .. \quad .. \quad (4.9)$$

In (4.7) we recognise in p_{AB} the term representing the single reflections within the line; in p_G the first-order reflected wave due to the mis-match between the generator and line; in $p_T \exp(-2P_{AB})$ the reflection from the mis-match between termination and line, modified by the propagation coefficient due to a double traverse of the line; and in $p^2_G p_{AB}$ the third-order reflection between generator and line which may be appreciable if the mis-match is large. In (4.9) we distinguish q_{AB} due to double reflections within the line; $p_{AB}p_G$ representing single reflections in the line reflected again at the generator; $p_{BA}p_T$ representing reflection from the termination reflected again in the line; and $p_G p_T \exp(-2P_{AB})$ representing double reflection between termination and generator.

If $Z_G = Z_T$ and the mismatch between line and terminal impedances is small, then

$$p_{GABG} = p_{AB} + p_G\{1 - \exp(-2P_{AB})\} \quad (4.10)$$

$$m_{GABG} = m_{AB} - p^2_G - p_G(p_{AB} + p_{BA}) + p^2_G \exp(-2P_{AB}) \quad .. \quad .. \quad (4.11)$$

$$q_{GABG} = q_{AB} - p_G(p_{AB} + p_{BA}) + p^2_G \exp(-2P_{AB}) \quad .. \quad .. \quad (4.12)$$

4.3. *Irregular Line with Small Impedance Deviations*

For a nominally homogeneous line OL of length l, we may define the average characteristic impedance Z_m as

$$Z_m = \frac{1}{l}\int_0^l Z_x dx \quad .. \quad .. \quad .. \quad (4.13)$$

Thus we may write

$$Z_x = Z_m + S_x \quad .. \quad .. \quad .. \quad (4.14)$$

$$= Z_r + \sigma_x \quad .. \quad .. \quad .. \quad (4.15)$$

where Z_r is a reference impedance sufficiently close to Z_m to make $(Z_r - Z_m)^2/Z_m{}^2$ negligible compared with 1; σ_x is the local deviation from this reference impedance, and S_x is the particular value of the deviation when the reference impedance is equal to the mean impedance.

Similarly for the nominally homogeneous line the average propagation coefficient per unit length, γ, is defined as

$$\gamma = \frac{1}{l}\int_0^l \gamma_x dx \quad .. \quad .. \quad .. \quad (4.16)$$

and $$\gamma_x = \gamma + g_x \quad .. \quad .. \quad .. \quad (4.17)$$

If σ_x/Z_x is small, then g_x/γ_x will be smaller. This follows because the principal deviations are due to inductance and capacitance variations, and these in turn are produced mainly by variations in conductor dimensions and in effective permittivity. The former produce local changes in L and C which to a first approximation add for Z_x but cancel for γ_x, and it is only changes in the dielectric which affect both Z_x and γ_x to the same degree.

Considering the line with one deviating portion discussed in Section (3.4), as the length of AB becomes progressively smaller and in the limit approaches dx, the propagation coefficient P_{AB} becomes $\gamma_x dx$. The reflection and transmission coefficients from (4.1) and (4.2) become

$$p_{AB} = 2p_x\gamma_x dx = 2p_x\gamma dx \text{ (since } g_x/\gamma \text{ is small)} \quad (4.18)$$

$$m_{AB} = 1 - 2p^2{}_x\gamma dx \quad .. \quad .. \quad (4.19)$$

where $$p_x = \sigma_x/2Z_r \quad .. \quad .. \quad .. \quad (4.20)$$

An irregular line is composed of a large number of adjacent small portions such as AB, each contributing a term $2p_x \exp(-2P_{ox})dx$ to the reflection coefficient. Replacing P_{ox} by γx introduces a negligible error when g_x/γ is small, and thus for the irregular line, we have from (4.3)

$$p_r = \frac{\gamma}{Z_r}\int_0^l \sigma_x \exp(-2\gamma x)dx \quad .. \quad (4.21)$$

where p_r denotes the reflection coefficient with respect to Z_r. When the mean impedance is selected as reference impedance, p is written without a suffix as this is a frequent case, and we have

$$p = \frac{\gamma}{Z_m}\int_0^l S_x \exp(-2\gamma x)dx \quad (4.22)$$

$$= p_r + \frac{\Delta Z}{2Z_m}\{1 - \exp(-2\gamma l)\} \quad (4.23)$$

where $\Delta Z = Z_r - Z_m$.

Similarly for the transmission coefficient, we have from (4.6)

$$m_r = 1 - \frac{\gamma}{2Z^2{}_r}\int_0^l \sigma^2{}_x dx + \frac{\gamma^2}{Z^2{}_r}\int_0^l \sigma_x \exp(-2\gamma x)\int_0^x \sigma_r \exp(2\gamma y)dy\,dx \quad .. \quad .. \quad .. \quad (4.24)$$

$$\text{or } m = 1 - \frac{\gamma}{2Z^2{}_m}\int_0^l S^2{}_x dx + \frac{\gamma^2}{Z^2{}_m}\int_0^l S_x \exp(-2\gamma x)\int_0^x S_r \exp(2\gamma y)\,dy dx \quad .. \quad .. \quad .. \quad (4.25)$$

To appreciate the physical basis of the integral forms of the expressions for the reflection and transmission coefficients, we revert to the line built up of a number of adjacent deviating portions. From this it is apparent that equation (4.21) contains the reflection from a given point as the difference of two components, viz., the ordinates of the infinitesimal portions on either side of the point. For example, in considering the 'tail' of a television signal,[3,5] one computes the reflection coefficient due to irregularities situated at distances greater than a given value from the origin. If we divide p_r as given by equation (4.21) into two parts representing reflection from O to B ($x = b$) and from B to L, Fig. 7, we obtain

$$p_{OB} = \frac{\gamma}{2Z_r}\int_0^b \sigma_x \exp(-2\gamma x)dx \qquad .. \quad (4.26a)$$

$$p_{BL} = \frac{\gamma}{2Z_r}\int_b^l \sigma_x \exp(-2\gamma x)dx \qquad .. \quad (4.26b)$$

and have thereby added unnecessarily $-\sigma_B \exp(-2\gamma b)$ to equation (4.26a) and $+\sigma_B \exp(-2\gamma b)$ to equation (4.26b). For a physical division of the line, with termination of each part by Z_r, equations (4.26a) and (4.26b) would of course be correct. The difference is brought out by throwing equation (4.21) into an alternative form by integrating it by parts. Thus we obtain

$$p_r = \frac{1}{2Z_r}\left[\int_0^l \sigma'_x \exp(-2\gamma x)\,dx + \sigma_o - \sigma_L \exp(-2\gamma l)\right] \quad (4.27)$$

where $\sigma'_x = \dfrac{\partial \sigma_x}{\partial x}$ and σ_o, σ_L are the values of the impedance deviations at the beginning and end of the line. On breaking up the reflection coefficient, the two parts corresponding to equations (4.26a) and (4.26b) are

$$p_{OB} = \frac{1}{2Z_r}\left[\sigma_o + \int_0^b \sigma'_x \exp(-2\gamma x)\,dx\right] \quad (4.28a)$$

$$p_{BL} = \frac{1}{2Z_r}\left[\int_b^l \sigma'_x \exp(-2\gamma x)dx - \sigma_L \exp(-2\gamma l)\right] \quad .. \quad (4.28b)$$

and in this form no unnecessary terms are introduced. On the other hand, these expressions are not correct if the line is physically divided and each section is terminated by Z_r. To appreciate this aspect, we notice that in equation (4.27) the term $\sigma_o/2Z_r$ represents the reflection coefficient for the step between the generator and the entrance to the line, and $\sigma_L \exp(-2\gamma l)/2Z_r$ the reflection coefficient for the step between the end of the line and the terminal impedance.

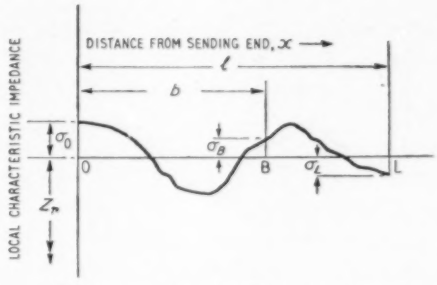


Fig. 7. *Variation of impedance along irregular line.*

Thus $\dfrac{1}{2Z_r}\int_0^l \sigma'_x \exp(-2\gamma x)\,dx$ represents the reflection coefficient of the line when the generator and terminal impedances are respectively equal to the impedances of the portions of line immediately adjacent to them. With an irregular line, these impedances are in practice indeterminate, as they depend on the length over which the impedance may be regarded as uniform, so that for measurement purposes it is usually preferable to use the form of equation (4.21).

Similar considerations apply to the integral form of the equation for the transmission coefficient. In equation (4.24) the interaction between adjacent infinitesimal portions is contained partly in the first term and partly in the second; furthermore the division into parts again introduces unnecessary terms. This may be avoided by throwing (4.24) into an alternative form by integrating by parts, when we obtain

$$m_r = 1 + \frac{1}{4Z_r^2}\left[\sigma_o\sigma_L \exp(-2\gamma l) - \tfrac{1}{2}(\sigma_o^2 + \sigma_L^2) - \sigma_o\int_0^l \sigma'_x \exp(-2\gamma x)dx + \sigma_L\int_0^l \sigma'_x \exp\{-2\gamma(l-x)\}\,dx - \int_0^l \sigma'_x \exp(-2\gamma x)\int_0^x \sigma'_r \exp(2\gamma y)dy\,dx\right] \quad (4.29)$$

Here the term $-\dfrac{1}{4Z_r^2}\int_0^l \sigma'_x \exp(-2\gamma x)\int_0^x \sigma'_r \exp(2\gamma y)dy\,dx$ represents the

transmitted echo coefficient when the generator and termination equal the respective impedances of the portions immediately adjacent to them.

Equation (4.27) may also be put into the form

$$p_T = \frac{1}{2Z_T} \sum_{x=0}^{x=L} \exp(-2\gamma x)\delta Z_x \quad .. \quad .. \quad (4.30)$$

where δZ_x is the change of impedance at X, and the sum includes the steps at the beginning and end of the line. In this form, the equation is useful when we wish to consider the effects of abrupt changes of impedance.

5. Effect of Small Line Irregularities on the Open- and Short-Circuited Impedances

When a uniform line is open-circuited at its distant end, its input impedance is

$$Z_{op} = Z \coth \gamma l \quad .. \quad .. \quad .. \quad .. \quad (5.1)$$

When the distant-end is short-circuited, the input impedance becomes

$$Z_{sh} = Z \tanh \gamma l \quad .. \quad .. \quad .. \quad (5.2)$$

If a line AB has small irregularities and is terminated by an impedance Z_T, the input impedance is from equation (3.47)

$$Z_{AT} = Z_m \left[\frac{1 + p_{AB} + p_T \exp(-2\gamma l)/(1 - p_{BA}p_T)}{1 - p_{AB} - p_T \exp(-2\gamma l)/(1 - p_{BA}p_T)} \right] \quad .. \quad (5.3)$$

where p_{AB}, p_{BA}, are the reflection coefficients at A and B respectively with the distant end terminated by Z_m, and $p_T = (Z_T - Z_m)/(Z_T + Z_m)$. When the distant end is open, $p_T = +1$, and when it is closed $p_T = -1$. Thus the input impedance of an irregular line with the distant end open is

$$Z_{Aop} = Z_m \left[\frac{1 + p_{AB} + \exp(-2\gamma l)/(1 - p_{BA})}{1 - p_{AB} - \exp(-2\gamma l)/(1 - p_{BA})} \right] \quad .. \quad .. \quad .. \quad .. \quad (5.4)$$

When the distant end is closed, the input impedance becomes

$$Z_{Ash} = Z_m \left[\frac{1 + p_{AB} - \exp(-2\gamma l)/(1 + p_{BA})}{1 - p_{AB} + \exp(-2\gamma l)/(1 + p_{BA})} \right] \quad .. \quad .. \quad .. \quad .. \quad (5.5)$$

Provided $p_{AB} + p_{BA}$ is small compared with $1 - \exp(-2\gamma l)$, which is generally the case,

$$Z_{Aop} = Z_m \coth \gamma l \left[1 + \frac{p_{AB} + p_{BA}}{1 - \exp(-2\gamma l)} + \frac{p_{AB} - p_{BA}}{1 + \exp(-2\gamma l)} \right] \quad (5.6)$$

Similarly, provided $p_{AB} - p_{BA}$ is small compared with $1 + \exp(-2\gamma l)$,

$$Z_{Ash} = Z_m \tanh \gamma l \left[1 + \frac{p_{AB} + p_{BA}}{1 - \exp(-2\gamma l)} + \frac{p_{AB} - p_{BA}}{1 + \exp(-2\gamma l)} \right] \quad (5.7)$$

Thus a correction factor K_{AB} which gives the influence of irregularities on the open- and short-circuited input-impedances of the corresponding uniform line, is

$$K_{AB} = 1 + \frac{p_{AB} + p_{BA}}{1 - \exp(-2\gamma l)} + \frac{p_{AB} - p_{BA}}{1 + \exp(-2\gamma l)} \quad .. \quad .. \quad (5.8)$$

Equivalent formulae are quoted by Kaden,[2] and by Parcé,[7] but proofs are not given. Thus under these conditions, the apparent characteristic impedance is

$$Z' = \sqrt{Z_{Aop} Z_{Ash}} = Z_m K_{AB} \quad .. \quad .. \quad (5.9)$$

The apparent propagation coefficient is

$$P' = \tanh^{-1} \sqrt{Z_{Ash}/Z_{Aop}} = \gamma l \quad .. \quad .. \quad (5.10)$$

Hence when measuring by the 'open and closed' method, small line irregularities would have an appreciable effect on the apparent characteristic impedance, but none on the attenuation and phase-change coefficients. Caution is necessary, however, in applying the last conclusion to measurements made when the line is in resonance.

6. Conclusion

The Kennelly method of successive reflections has been shown to yield expressions by simple mathematical processes which are in agreement with those derived by more formal methods. The relative coefficients may be calculated and measured, and the results applied to the evaluation of the quality of irregular lines for the transmission of broad-band telephony and television, but the discussion of these topics is outside the scope of this paper.

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THE E-H SURFACE WAVE

By A. E. Karbowiak

SUMMARY—The basic formulae for the E-H surface wave (which is a mixture of two surface waves; the E- and H- modes) are given. The cylindrical guide that is conducting in a helical direction is analysed in detail and the canonical frequency equation is put into a very simple form by making suitable approximations. Finally, it is shown, that although a travelling E-H surface wave can be supported by a helically-conducting guide, it is not possible to have a pure E-H surface wave on a transmission line terminated by a short-circuiting plate. Yet two similar E-H surface waves travelling in opposite directions on the same line give rise to an elliptically-polarized surface wave.

1. Introduction

THE wave mode usually associated with a single-wire transmission line[1] is the E-mode that is evanescent over the wave front. This is a form of electromagnetic wave which propagates without radiation along the outside of a single cylindrical guide[6] and is characterized by the presence of an axial component of the electric field vector and absence of the longitudinal component of the magnetic field vector (E-mode). In the plane normal to the axis of the guide the electromagnetic field extends to infinity in the radial direction, but the energy density of the field decays monotonically with radial distance at a rate determined by the surface properties of the guide. At large distances from the guide the field quantities decay approximately exponentially and usually most of the energy of the wave is constrained to flow in the immediate neighbourhood of the guide. Such a field is called evanescent. The surface wave on the *outside* of the guide is often accompanied by a guided wave *inside* the guide which is not a surface wave but whose nature is a function of the physical configuration of the guide structure, and is irrelevant except in so far that it provides the necessary guide surface impedance. The performance of a single-wire transmission line is governed by its radial decay coefficient, and this depends solely on the guide radius and the surface impedance.[2,3] Of course, it is assumed that most of the power is carried on the outside and scarcely any on the inside. This $E_{0\times}$ mode (where $\times$ signifies evanescent nature of the wave in the radial direction) has been investigated in detail[2,3], and there are at least two distinct methods of decreasing the lateral spread of the field (i) by coating the wire with a layer of dielectric and (ii) by corrugating the surface of the guide.

It has been known for some time that a helix made of a conducting wire will support a wave mode which has found particular application in the travelling-wave valve[4]. The wave associated with this structure possesses axial components of both electric and magnetic field, and since the wave is evanescent over the wave front on the outside of the helix we shall call the wave mode exterior to the helix the E-H surface wave ($EH_{0\times}$mode). Kaden[5] recognized the bearing of this type of wave on possible future development in ultra-high-frequency transmission lines and suggested the use of a stranded conductor for single-wire transmission lines. It is the purpose of this paper to present the basic formulae of the E-H surface wave and to investigate some of its characteristics when supported by a helically-conducting guide.

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2. Field Equations of the Progressive E-H Surface Wave

Let us describe the electromagnetic field in terms of the cylindrical co-ordinates: r, ϕ, z (Fig. 1). Assuming negligible losses the field outside the cylindrical guide radius s is derived from the wave function*

$$F_1(r,z) = \mathrm{H}_0^{(1)}(jur) \cdot e^{j(\omega t - \beta z)} \quad .. \quad (1)$$

where $\beta^2 - u^2 = h^2 = \omega^2\mu_0\kappa_0$ .. .. (2)

and u = radial propagation coefficient

β = axial propagation coefficient

$\mathrm{H}_n^{(1)}$ = Hankel function of order n

μ_0, κ_0 = permeability and permittivity of free-space respectively, in m.k.s. system of units.

Omitting the factor $e^{j(\omega t - \beta z)}$ the field components of the E-H surface wave become

$$\left.\begin{aligned} E_{z1} &= A_1\mathrm{H}_0^{(1)}(jur) \\ H_{z1} &= B_1\mathrm{H}_0^{(1)}(jur) \\ E_{r1} &= A_1\frac{\beta}{u}\mathrm{H}_1^{(1)}(jur) \\ H_{r1} &= B_1\frac{\beta}{u}\mathrm{H}_1^{(1)}(jur) \\ H_{\phi 1} &= A_1\frac{\omega\kappa_0}{u}\mathrm{H}_1^{(1)}(jur) \\ E_{\phi 1} &= -B_1\frac{\omega\mu_0}{u}\mathrm{H}_1^{(1)}(jur) \end{aligned}\right\} \quad .. \quad .. \quad (3)$$

where A_1, B_1 are two independent constants.

Inside the guide (radius s) the electromagnetic

* Stratton, J. A. "Electromagnetic Theory" (McGraw-Hill), 1941, p. 360. Schelkunoff, S. A. "Electromagnetic Waves" (D. Van Nostrand Co.), 1943, p. 406.

field is derived from the wave function

$$F_2(r_1 z) = \mathrm{J}_0(hr) \cdot e^{j(\omega t - \beta z)} \quad .. \quad .. \quad (4)$$

where $\beta^2 + h^2 = k_1^2 = \omega^2 \mu_1 \kappa_1$ .. .. (5)

and h = radial propagation coefficient

μ_1, κ_1 = permeability and permittivity of the dielectric inside the guide

J_n = Bessel function of order n.

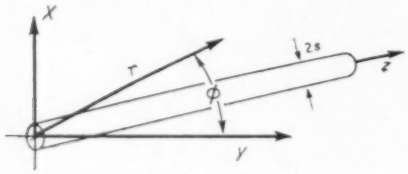

Fig. 1. *The co-ordinate system.*

The field components inside the guide will thus become (the exponential factor is understood);

$$\left.\begin{aligned} E_{z2} &= A_2 \mathrm{J}_0(hr) \quad , \\ H_{z2} &= B_2 \mathrm{J}_0(hr) \\ E_{r2} &= A_2 \frac{j\beta}{h} \mathrm{J}_1(hr) \quad , \\ H_{r2} &= B_2 \frac{j\beta}{h} \mathrm{J}_1(hr) \\ H_{\phi 2} &= A_2 \frac{\omega j \kappa_0}{h} \mathrm{J}_1(hr) \quad , \\ E_{\phi 2} &= -B_2 \frac{j\omega\mu}{h} \mathrm{J}_1(hr) \end{aligned}\right\} \quad .. \quad .. \quad (6)$$

where A_2, B_2 are two different constants.

The particular relation between the constants A_1, B_1 and A_2, B_2 will depend on the nature of the guide but, whatever these constants are, the equations (3) represent the field of an E-H surface wave provided u lies in the fourth quadrant of the Argand diagram.[2,3] If $B_1 = 0$ then the remaining equations of (3) represent a pure E-surface wave and if $A_1 = 0$ then (3) represents an H-surface wave. It is essential for the existence of the H-surface wave that the lines of magnetic field strength shall penetrate the guide surface. The field thus can be evanescent on the outside of the guide and yet lines of magnetic field strength form closed loops in the plane containing the guide axis.

3. The Helically-Conducting Guide

Suppose that the guide is conducting in the helical direction ψ as shown in Fig. 2, and non-conducting in the direction normal to this. Thus the boundary conditions to be imposed on (3) and (5) are:—

(i) The tangential electric field must be continuous across the guide surface; i.e.,

$$\left.\begin{aligned} E_{z1} &= E_{z2} \quad (r = s) \\ E_{\phi 1} &= E_{\phi 2} \quad (r = s) \end{aligned}\right\} \quad .. \quad (7)$$

(ii) There is no electric field strength at the surface of the guide in the conducting direction; thus

$$\left.\begin{aligned} E_{z1} \sin\psi + E_{\phi 1} \cos\psi &= 0 \quad (r = s + 0) \\ E_{z2} \sin\psi + E_{\phi 2} \cos\psi &= 0 \quad (r = s - 0) \end{aligned}\right\} \quad (8)$$

(iii) The component of magnetic field strength in the direction of the helix must be continuous across the guide surface; thus

$$H_{z1} \sin\psi + H_{\phi 1} \cos\psi = H_{z2} \sin\psi + H_{\phi 2} \cos\psi \quad (r = s) \quad .. \quad (9)$$

Relations (7), (8) and (9) lead to four independent equations involving the coefficients A_1, B_1 and A_2, B_2 wherefrom upon elimination we obtain the equation

$$\frac{\mathrm{H}_0^{(1)}(jus)\mathrm{J}_0(hs)}{\mathrm{H}_1^{(1)}(jus)\mathrm{J}_1(hs)} \times \frac{uh}{j\omega^2\mu_1\kappa_1} = \chi \cot^2\psi \quad .. \quad (10)$$

where

$$\chi = \frac{us\dfrac{\mathrm{H}_0^{(1)}(jus)}{\mathrm{H}_1^{(1)}(jus)} + j\dfrac{\kappa_0}{\kappa} . hs. \dfrac{\mathrm{J}_0(hs)}{\mathrm{J}_1(hs)}}{us\dfrac{\mathrm{H}_0^{(1)}(jus)}{\mathrm{H}_1^{(1)}(jus)} + j . hs. \dfrac{\mathrm{J}_0(hs)}{\mathrm{J}_1(hs)}} \quad .. \quad (11)$$

and, from (2) and (5)

$$h^2 + u^2 = \omega^2\mu(\kappa_1 - \kappa_0) \quad .. \quad .. \quad (12)$$

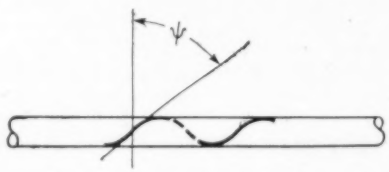

Fig. 2. *Cylindrical guide conducting in the direction of a helix.*

Thus, given the radius of the guide and the lay of the helix then from equations (10)–(12) the decay coefficient u could in principle be calculated; however, it is a laborious procedure. If $\kappa_1 = \kappa_0$ as in a travelling-wave valve[4] then $\chi = 1$ and (10) could be solved by graphical means. But if the guide were made by winding a bunch of wires in a helical manner around the circumference of a dielectric rod then, of course, $\kappa_1 \neq \kappa_0$ and a different approach would be necessary. The expression (10) simplifies notably if we assume that the guide radius is small and that the decay coefficient is small in comparison with h. We then obtain $\chi = \kappa_0/\kappa_1$ and the frequency equation (10) becomes

$$\frac{\mathrm{H}_0^{(1)}(jus)\mathrm{J}_0(hs)}{\mathrm{H}_1^{(1)}(jus)\mathrm{J}_1(hs)} uh = jk_0^2 \cot^2\psi \quad .. \quad (13)$$

This equation could be solved for u graphically

but at this stage further approximations are legitimate. We may take

$$h \approx h_0 = \omega\sqrt{\mu(\kappa_1 - \kappa_0)} \quad .. \quad .. \quad (14)$$

and replace the Hankel functions occurring in (13) by their small argument approximations :

$$\left.\begin{aligned} \text{Thus :—} \quad H_0^{(1)}(jus) &\approx j\frac{2}{\pi}\log_e(0{\cdot}89us) \\ H_1^{(1)}(jus) &\approx -\frac{2}{\pi us} \end{aligned}\right\} \quad (15)$$

With these simplifications (13) reduces to

$$\xi \log_e \xi = -(0{\cdot}89)^2 \frac{J_1(h_0 s)}{J_0(h_0 s)} \cdot \frac{(k_0 s \cot\psi)^2}{h_0 s} \quad .. \quad (16)$$

where[2],[3] $\xi = (0{\cdot}89us)^2$

Now, the fundamental equation is[2]

$$\xi \log_e \xi = \eta \quad .. \quad .. \quad .. \quad .. \quad (17)$$

where

$$\eta = -2{\cdot}(0{\cdot}89)^2 k_0^2 s X_s \quad .. \quad .. \quad (18)$$

X_s = surface reactance of a guide carrying an E surface wave

and

$$Z_0 = \sqrt{\mu_0/\kappa_0} = 377 \text{ ohms.}$$

Thus, equating (16) and (18) we get

$$X_s = \frac{1}{2}\frac{Z_0}{\sqrt{1 - \kappa_0/\kappa}} \cdot \frac{J_1(h_0 s)}{J_0(h_0 s)} \cot^2\psi \quad (\kappa_0 \neq \kappa_1) \quad .. \quad (19)$$

and

$$X_s = \frac{Z_0}{4} k_0 s \cot\psi \quad \text{if } \kappa_1 = \kappa_0 \quad .. \quad .. \quad (20)$$

We shall refer to (19) and (20) as the equivalent surface reactance of a helically-conducting guide. These formulae are simple to use and curves shown in Fig. 3 were calculated using equation (20).

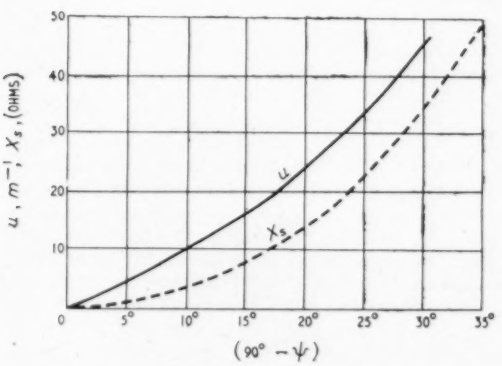

Fig. 3. *The radial propagation coefficient u and the surface reactance X_s as a function of the lay angle ψ.*

4. The E-H Surface Wave other than Progressive

In Section 2 we have established the possible existence of a progressive E-H surface wave. In practical applications, however, more often than not a partial standing wave will be encountered. We shall next enquire what is the field distribution of an E-H surface wave supported by a transmission line short-circuited at one end. Suppose the short-circuit occurs at $z = 0$ then the boundary condition to be satisfied at this point

$$\text{is } E_r(r,0) = E_\phi(r,0) = 0 \quad .. \quad .. \quad .. \quad (21)$$

while the boundary condition to be satisfied at the surface of the guide is

$$E_{\phi_1}(s, z) = \tau E_{z1}(s, z) \quad .. \quad .. \quad .. \quad (22)$$

where τ is a factor depending on the nature of the guide.* It is not difficult to show that the boundary conditions (21) and (22) cannot be simultaneously satisfied. The inference can be drawn that a *pure* E-H surface wave may exist as a travelling wave, but is not possible on a short-circuited transmission line. The physical reason for this is that a 'right-handed' E-H surface wave (surface wave supported by a right-handed helix) is reflected from a short-circuiting plate as a 'left-handed' E-H surface wave (capable of being supported by a left-handed helix) and this cannot be supported by a right-handed helical guide necessary for the support of the outgoing wave.

Nevertheless it is possible, at least mathematically, to have two right-handed/left-handed E-H surface waves travelling in opposite directions on a right-handed/left-handed helical guide, but the resultant field does not possess the character of a standing wave (where the transverse component of the electric vector exhibits successive nodes and anti-nodes). This field is an elliptically polarized E-H surface wave whose plane of polarization is turning clockwise/anti-clockwise with respect to an observer moving along the line.

Acknowledgments

The author acknowledges gratefully helpful discussions with Professor H. M. Barlow and Dr. A. L. Cullen of University College, London.

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* In the case of the helical guide it takes the form given by (8); i.e., $\tau = \tan\psi$.

TAPPED INDUCTANCES

Calculation of Tapping Points

By C. R. Cosens, M.A.

INDUCTORS for use at audio-frequencies are conveniently wound as 'Maxwell coils'; i.e., they are circular coils of square cross-section, such that the inner diameter of the coil is twice the side of the square. It is sometimes desirable to provide tappings at specified fractions of the total inductance, and it is the object of this note to derive a simple method of calculating at what fraction n of the total turns a tapping must be made in order that the inductance measured between the inner end of the winding and the tapping shall be a specified fraction l of the inductance of the whole Maxwell coil. The simple formulae derived give an accuracy of 3 parts in 1,000, without reference to tables; they are based on equations and tables in Chapter 13 of F. W. Grover's "Inductance Calculations," Van Nostrand, New York, 1946.

From equation (94) of this chapter we find that the inductance L_M (microhenrys) of a complete Maxwell coil of width b (cm), having N_M turns, is given by:

$$L_M = 0{\cdot}02549\ N_M^2 \times b \quad .. \quad .. \quad .. \quad (1)$$

whence, putting τ = number of turns per sq. cm. of cross-section,

$$b = 2{\cdot}083 \times (L_M/\tau)^{1.5} \quad .. \quad .. \quad .. \quad (2)$$

and then $N_M = \tau b^2$ .. .. .. (3)

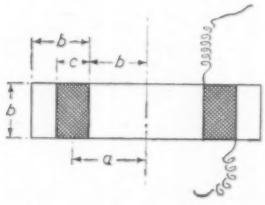


Fig. 1. *Sectional view of coil considered; full Maxwell coil shown in outline.*

The inductance of any circular coil of rectangular cross-section can be found in terms of the three dimensions a, b and c shown in Fig. 1, and of the turns N. For the special case of part of a Maxwell coil, from the inner end to the tapping, $2a = 2b + c$, or $2a/b = 2 + c/b$ (since the inner diameter of the coil is $2b$). The turns $N = \tau bc$, but for the whole Maxwell coil, to which the suffix $_M$ refers, $c_M = b$, so $N_M = \tau b^2$, hence $n = N/N_M = c/b$. It follows that the ratios

$$c/b = n,\ 2a/b = (n+2),\ \text{and}\ c/2a = n/(n+2) \quad (4)$$

MS accepted by the Editor, May 1953

can all be expressed in terms of the single variable n.

In terms of these ratios, Grover's Table 36 gives K ('Nagaoka's constant'), which takes account of the axial separation of the turns; and double-entry tables 22-25 give k, which takes account of their radial separation. Tabulating, to four decimals, to argument $n = 0{\cdot}0\ (0{\cdot}1)\ 1{\cdot}3$, these ratios, K, k, and $K' = K - k$, we have, from equation 99 (omitting a constant multiplier)

$$L \propto N^2 \times \left(2\frac{a}{b}\right) \times a \times K'$$

$$\text{or } \frac{2L}{b} \propto N^2 \times \left(2\frac{a}{b}\right)^2 \times K' = N^2[(n+2)^2 K'] \quad (5)$$

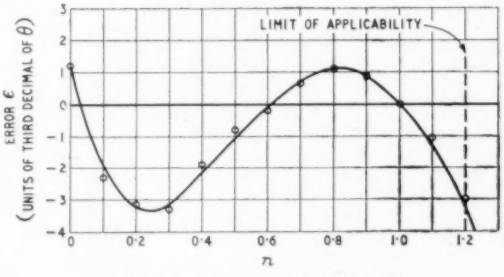


Fig. 2. *Error curve for the value of* θ.

For the complete Maxwell coil the square bracket [ ] is a constant, say β so that equation (5) gives $2L_M/b \propto N_M^2\beta$, and hence

$$l = \frac{L}{L_M} = \left(\frac{N}{N_M}\right)^2 \times [(n+2)^2 K']/\beta = n^2\phi, \quad (6)$$

say, where the numerical value of β is to be determined so that $\phi = [(n+2)^2\ K']/\beta$ shall equal 1 when $n = 1$, $l = 1$.

After some tedious computing we have a table of ϕ to argument n. But no table is satisfactory unless it can easily be interpolated; tables relating ϕ to l, n or n^2 all offer interpolation difficulties near zero, where the differences are not only large, but apparently divergent. The trouble is overcome if instead of ϕ we tabulate its reciprocal $\theta = 1/\phi$ to argument n. Having tabulated to four decimals, we round θ to three, and find, somewhat surprisingly, that we only incur errors between $+1$ and -3 in the third decimal place if we write

$$\theta = 1 + 0{\cdot}230\,(1-n), \text{ where } 0 < n < 1{\cdot}2;\ 0 < l < 1{\cdot}5 \quad (7)$$

Since over this range $1{\cdot}23 > \theta > 0{\cdot}98$, the error is less than 3 in 1,000.

Then $l = n^2/\theta$, or $n = \sqrt{l\theta}$ .. .. (8)

When seeking n for given l, we can avoid the solution of a cubic in n by using the method of iteration, thus:

$$n_0 = \sqrt{l},\ \theta_r = 1 + 0{\cdot}230(1 - n_r),\ n_{r+1} = \sqrt{l\theta_r}\ (9)$$

Convergence is rapid, n_4 nowhere differs from n_3 by more than half a unit in the third decimal place.

(Values of n and l greater than unity obviously refer to a coil having additional turns wound over and above those of a true Maxwell coil, this may sometimes be convenient.)

It may be of some slight (though purely academic) interest, to plot the errors of the empirical formula $\theta = 1 + 0{\cdot}230\ (1 - n)$. This is shown in Fig. 2 where

ϵ = (empirical) — (computed from Grover).

Since only four decimals and linear interpolation in Grover's tables were written throughout, plotted points may be expected to be unreliable within about $\pm 0{\cdot}03$.

The shape of the curve is strongly reminiscent of a cubic, with zero quadratic term. This suggests the possibility that if θ, defined as n^2/l, could be expanded in ascending powers of n, from the theoretical formulae used by Grover to compute his tables (they are not specified, presumably combinations of elliptic integrals) the coefficient of n^2 in the expansion $\theta = a + bn + cn^2 + dn^3 + \ldots$ would be found to vanish. This would explain the unexpected closeness of the linear approximation $\theta \approx 1 + 0{\cdot}230\ (1 - n)$ (better than 3 parts in 1,000). Admittedly this is a shot in the dark.

For practical purposes, of course, since the number of turns will not, in general, form complete layers, as the formula assumes, accuracy better than $\pm 1\%$ would be quite illusory.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Output Impedance Theorem

SIR,—I have noticed the great interest of *Wireless Engineer* in new circuit theorems, and I therefore submit the following:—

In the general case when a constant-voltage generator, c.v.g. or e.m.f., or a constant-current generator, c.c.g., in the equivalent circuit described by Thévenin or Norton, is independent of a voltage applied to the terminals AB of the equivalent circuit, the output impedance is defined as the inner impedance, with the added series or parallel impedance, contributed by the equivalent c.v.g. or c.c.g., interpreted as an impedance with the aid of the compensation theorem.

When the internal c.v.g. or c.c.g. is a function of the applied voltage V_{AB}, and proportional to it, this relation may be expressed via a known proportionality constant k. It is then possible to write down directly the output impedance Z_{AB}, if use is made of a new theorem, which has the form:

Theorem: If in a linear, active, one-pair terminal network, described by Thévenin's or Norton's theorems, the c.v.g. or the series injected voltage from the c.c.g., is proportional to the voltage V_{AB} applied to the terminals AB, and k times V_{AB}, the output impedance is the passive network output impedance, divided by $(k + 1)$.

A passive network obtains when all c.v.gs are short-circuited, and all c.c.gs open-circuited. This theorem makes unnecessary the in-between application of the compensation theorem (which application is usually made unconsciously, anyhow), and yields the answer more or less directly.

As an example, consider a simple cathode follower, Fig. 1(a), with its series equivalent circuit in Fig. 1(b), which is a Thévenin circuit. Since $k = \mu$, the above theorem states that $Z_{AB} = r_a/(\mu + 1)$, which is the correct result. If the parallel equivalent circuit for a cathode follower is used or, alternatively, the circuit used that results when the voltage source in Fig. 1(b) is represented as a current source, the Norton circuit in Fig. 1(c) results. Then the 'series injected' voltage from the c.c.g. is $(\mu V_{AB}/r_a) \times r_a = \mu V_{AB}$, thus $k = \mu$, and as before $Z_{AB} = r_a/(\mu + 1)$. Since in many feedback

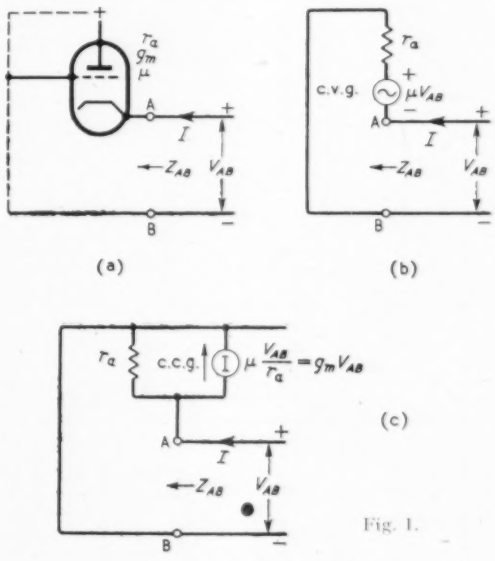


Fig. 1.

circuits k, or the general form of k, is known, Z_{AB} can be written down upon inspection, not necessitating calculation.

With reference to this theorem, I take the liberty to call to general attention the unfortunate fact that not only are there about as many formulations of Thévenin's and Norton's theorems as there are languages, but within each language, such as the English, there are about a dozen different wordings of each theorem. In so far as different languages permit, not only could one single formulation of each theorem be agreed upon, but actually *only one formulation for both of them*. Thus the "Thévenin-Norton theorem," (perhaps in the future called "*the General Theorem of the Equivalent Source*"), would read:

> *Theorem*: A linear, active, one-pair terminal network providing between its terminals AB the open circuit voltage V_{AB}, or short-circuit current I, may be replaced by an *equivalent source* of impedance Z_{AB}, which is the passive network looking-in impedance through the terminal pair, provided with the series c.v.g. V_{AB}, or shunt c.c.g. I.

There are other pairs of network theorems which may be 'internationalized,' standardized, and unified, into c.v.g.-c.c.g. formulation in accordance with the principle indicated above.

HARRY STOCKMAN.

Scientific Specialties Corporation,
Boston 35, Massachusetts, U.S.A.
28th December 1953.

A Circuit Problem

SIR,—In a letter by R. N. Bracewell,[1] which appeared in the January issue of this journal, attention was drawn to the following electric circuit problem: To connect the smallest possible number of 1-ohm resistors into a two-terminal network in such a way that no two resistors carry equal currents and so that the resistance of the combination is 1 ohm. The trivial case of one single resistor is excluded.

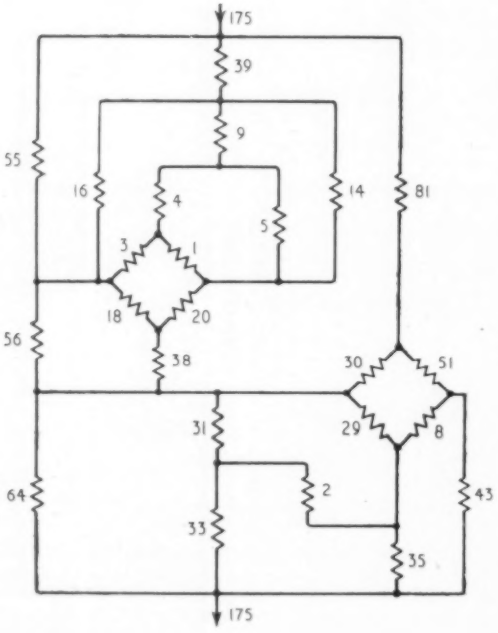


Fig. 1.

Although the solution of this curious problem is unknown to me, I should like to point out that it is not identical with the network given by Bracewell. His network, consisting of 26 resistors, is in fact a re-drawing of a planar graph equivalent to the perfectly squared square of Fig. 9 in a paper[2] cited by Bracewell, which at that time (1940) was the simplest known. Since then, T. H. Willcocks[3], of Bristol, has constructed such a square of order 24. The corresponding network is shown in Fig. 1, built up from only 24 resistors, that is, two less than in Bracewell's case.

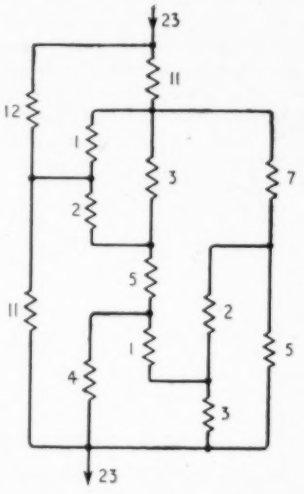


Fig. 2.

Some time ago I solved an analogous but simpler problem[4]: To connect the smallest possible number of 1-ohm resistors into a planar two-terminal network without series or parallel connections or zero currents so that the resistance of the combination is 1 ohm. This smallest number is 13. One of the two solutions to the second problem is shown in Fig. 2; the alternative is the dual of Fig. 2.

The numbers in Figs. 1 and 2 denote relative values of currents.

C. J. BOUWKAMP.

Philips Research Laboratories,
Eindhoven, Holland.
10th January, 1954.

[1] R. N. Bracewell, *Wireless Engr*, January 1954, Vol. 31, p. 27.

[2] R. L. Brooks, C. A. B. Smith, A. H. Stone and W. T. Tutte, *Duke math. J.* 1940, Vol. 7, pp. 312-340.

[3] T. H. Willcocks, *Fairy Chess Rev.*, Aug./Oct. 1948, Vol. 7, *Canad. J. Math.*, 1951, Vol. 3, pp. 304-308.

[4] C. J. Bouwkamp, *Proc. kon. ned. Akad. Wet. Amst.*, 1946, Vol. 49, pp. 1176-1188, Ibid., 1947, Vol. 50, pp. 58-78, pp. 1296-1299.

"WIRELESS ENGINEER" INDEXES

The annual indexes for 1953, both to *Wireless Engineer* articles and to Abstracts and References, are included in this issue. They are placed at the end so that by opening the staples they can readily be removed for binding with the 1953 Volume.

When binding, it is usually most convenient to separate the editorial and abstracts pages of each issue and to assemble them together in two blocks. The order of assembly for binding should then be: Index to Articles and Authors, all Editorial pages, all Abstracts pages, Index to Abstracts. It has been found that this grouping considerably facilitates reference.

NEW BOOKS

Probability and Information Theory, with Applications to Radar

By P. M. Woodward, B.A. Pergamon Press, Ltd., 242 Marylebone Road., London, N.W.1. Pp. 128 + x. Price 21s.

This book is one of the Pergamon Science series of monographs under the general title of 'Electronics and Waves' and under the general editorship of D. W. Fry of Harwell. Mr. Woodward is a mathematician in the Theoretical Physics division of the Telecommunications Research Establishment at Malvern. He is a well-known expert on communication theory, and has a marked ability to explain a point clearly and concisely in a lecture or discussion. The same clarity pervades much of the writing of the present book, which is intended to explain the theory by easy stages for the reader who has no highly advanced mathematical knowledge but wishes to link practical intuition and experience with mathematical theory. The illustrative numerical examples are excellent; by following them in detail, the reader can be sure that he has grasped the application of basic principles to artificially simple cases.

But in order that the book could be published quickly and at a reasonable price, it has had to be drastically condensed. It is a very fine achievement by Mr. Woodward to have produced at all a book of 125 pages which covers in essence so much ground. At the same time there is necessarily a hard core of mathematics for which this condensation may tend to decrease an engineer's initially insufficient mathematical confidence in himself. Key steps by which relevant results can be derived from Mr. Woodward's equations are indicated, but the significance of some equations is not easy to grasp. This is especially true where the equations are in an unfamiliar form because Mr. Woodward has neatly and usefully introduced certain new functions.

The first two chapters are concerned with probability theory, waveform analysis and noise. "They provide the code in which so much of the mathematical theory of electronics and radar is nowadays expressed." Chapter 3 is a summary of Shannon's original work[1]. Chapter 4 is concerned with the reception of a signal in noise by a communication system, while Chapter 5 is concerned with radar reception. Chapter 6, dealing with the mathematical analysis of radar information, is a revised account of work carried out by Mr. Woodward in partnership with Mr. I. L. Davies[2]. In Chapter 7 the transmitted radar signal is considered for a stationary target.

Information theory is by its essential nature a mathematical subject, and this fact must be faced by anyone interested; Mr. Woodward's book is a great help to those who are capable of facing it. J. W. H.

[1] "The Mathematical Theory of Communication", by C. E. Shannon and W. Weaver. Urbana: University of Illinois Press, 1949.
[2] *Phil. Mag.*, 1950, Vol. 41, p. 1001, and *Proc. Instn elect. Engrs* (Part III), 1952, Vol. 99, p. 37.

Fundamentals of Electronic Motion

By Willis W. Harman. Pp. 319 + x. McGraw-Hill Publishing Co., Ltd., 95 Farringdon Street, London, E.C.4. Price 46s. 6d.

In a lengthy preface the author, who is Associate Professor of Electrical Engineering at Stanford University, says that this is a book on analysis and that "the fact that it is written about electron tubes is to a certain extent incidental." Furthermore, he says that "Many educators feel that the answer to the problem imposed by the fabulously increasing body of engineering knowledge is increased concentration on general philosophies, understandings and attitudes, with correspondingly decreased emphasis on current engineering practice and specific design techniques. This book is written in that spirit." The author goes on to say that electron-tube analysis best demonstrates the philosophies and techniques of mathematical physics, for "particle dynamics, electromagnetic fields, and wave phenomena are all represented". His primary object is "to nurture the ability to deal with new problems and new situations".

He intends the book for the user of electron tubes; to give a general understanding rather than specialized information. The presentation is mathematical, but the author claims that no initial knowledge of mathematics beyond the calculus is necessary. Other mathematical techniques are explained as they are needed.

Chapter 1 covers the basic laws of electric and magnetic fields and their mathematical expression in the form of partial differential equations. These are in the curl, div, grad form and include the Laplace and Poisson equations. Approximate methods of field determination are also treated.

Chapter 2 deals with the motion of an electron in a static electric field and includes electron lenses. Chapter 3 is entitled "Electron Properties and Sources". Starting with the electron microscope, one proceeds through a discussion on electron diffraction, the electron as a wave packet and electron motion in solids to thermionic emission. Next, in Chapter 4, comes electron motion in a magnetic field, and in Chapter 5, negative and positive space charge. Chapters on motion in time-varying fields, space-charge waves and velocity modulation, travelling-wave amplification, magnetrons, and relativistic electrodynamics follow.

The book is not easy reading. It is one for study and it has its limitations although, within those limitations, it is very good. Its most valuable feature is the way in which problems are translated into mathematical form. This is where the novice usually runs into difficulty. Occasionally, a somewhat greater clarity of explanation would be desirable but, usually, this is all very well done.

It is also a good point that the solution of the resulting equations is carried out fairly fully. Since this is not a mathematical textbook, the author might have been forgiven for omitting this aspect. It is a fact, however, that it is much easier to learn mathematics in conjunction with an application of them than in the dreadfully arid atmosphere of the usual mathematics book. It is to the author's credit that he has evidently realized this.

On the whole, the author has done very well what he set out to do and has produced a very useful book. The student should be warned that in many cases things are not as simple as they are made to appear and one must disagree with the author's statement that the book is for "the user of electron tubes". In the ordinary meaning of these words the book is not for the user. It is for a rather specialized class of user who must know a great deal about what goes on inside a valve (and even this class will find it incomplete) but it is much more a basic textbook for the budding valve designer, who again will have to supplement it with a great deal of more technological material. W. T. C.

Relays for Electronic and Industrial Control

By R. C. Walker, B.Sc.(Lond.), A.M.I.Mech.E., A.M.I.E.E. Pp. 303 + xi. Chapman & Hall, Ltd., 37 Essex Street, Strand, London, W.C.2. Price 42s.

The opening words of the author's preface well explain the scope of this book; they are:—"The object of this book is to bring together in collected form for

handy reference the principal features and potentialities of relays as switching devices". Although quite a lot of information about the mechanical and electrical sides of actual relays is included, the book is not so much for the designer of relays as for the designer of equipment which incorporates them. In spite of this, there is a great deal of information about the magnetic circuit, forms of contact, temperature rise of coils and so on.

The treatment is descriptive rather than analytic and is largely non-mathematical, but not entirely so.

W. T. C.

Data and Circuits of Television Receiver Valves

By J. JAGER. Pp. 216 + xi. Book IIIc of "Electronic Valves" in the Philips Technical Library. Cleaver Hume Press Ltd., Wright's Lane, Kensington, London, W.8. Price 21s.

A major part of this book comprises data on valves, types ECC81, EF80, EB91, PL83, PL82, ECL80, PL81, PY80, PY81, PY82, EY51, and c.r. tubes, types MW36-44, MW36-24 and MW43-43. The data are of similar type to those given in valve-makers' catalogues but are much more comprehensive.

In addition, there are several pages of text for each valve in which its television applications are described and typical circuits are given. The practice described is continental, with f.m. sound and negative modulation, so that certain parts of it are inapplicable to British television, in particular the detector and video circuits.

Static Electrification

Pp. 104 + iv. The Institute of Physics, 47 Belgrave Square, London, S.W.1. Price 25s.

Twenty-four papers presented at a Symposium held by the Institute of Physics on 25th-27th March, 1953.

PREMIUMS FOR TECHNICAL WRITING

The Radio Industry Council has announced that six further premiums for technical writing have been awarded. They are of 25 guineas each to:—

G. G. Gouriet, A.M.I.E.E., for his article "Spectrum Equalization" which appeared in *Wireless Engineer*, May 1953.

A. W. Keen, M.I.R.E., A.M.I.E.E., for his article "Triode Transformation Groups", *Wireless Engineer*, October 1953.

A. H. Beck, B.Sc.(Eng.), A.M.I.E.E., and A. D. Brisbane, for their article "A Cylindrical Magnetron Ionization Gauge", *Vacuum*, April 1953.

D. McMullan, M.A., Ph.D., for his article "The Scanning Electron Microscope and the Electron-Optical Examination of Surfaces", *Electronic Engineering*, February 1953.

H. M. Davis, B.Sc., A.R.I.C., and Joyce Seaborn, B.Sc., for their article "A Linear Sweep Cathode-Ray Polarograph", *Electronic Engineering*, August 1953.

J. R. Pollard, M.A., M.I.E.E., M.I.R.E., for his article "Selective Calling for Radio-Telephone Systems", *Electronic Engineering*, December 1953.

The presentation of the awards is to be made at a luncheon of the Public Relations Committee of the Radio Industry Council on 11th March 1954.

MEETINGS

I.E.E.

10th March. "A Study of some of the Properties of Matter affecting Valve Reliability", by E. A. O'Donnell Roberts, M.Sc., Ph.D.

22nd March. "Colour Television", by C. J. Hirsch.

23rd March. "An Experimental and Theoretical Approach to the Teaching of Electromagnetism using the Rationalized M.K.S. System", by F. A. Meier, M.A., at 6 o'clock.

5th April. "Technical Problems involved in Receiving Alternative Television Programmes", discussion to be opened by F. R. W. Strafford and K. I. Jones.

These meetings will be held at the Institution of Electrical Engineers, Savoy Place, Victoria Embankment, London, W.C.2, and will commence at 5.30 except where otherwise stated.

Brit.I.R.E.

31st March. "Radio Astronomy", by R. Hanbury Brown, to be held at London School of Hygiene and Tropical Medicine, Keppel Street, Gower Street, London, W.C.2, commencing at 6.30.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

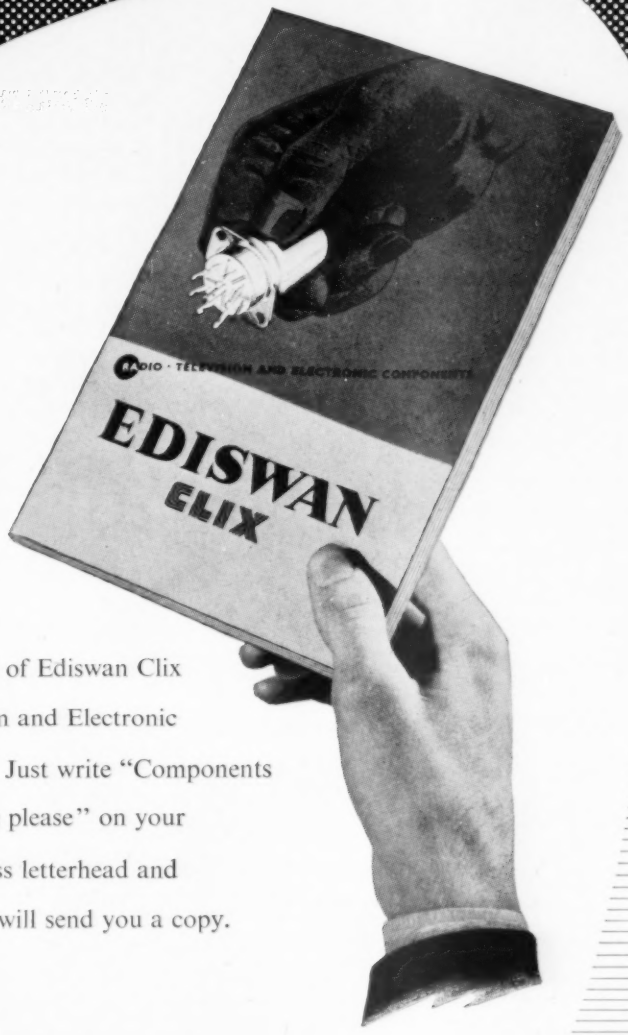
Values for January 1954

Date 1954 Jan.	Frequency deviation from nominal: parts in 10^8		Lead of MSF impulses on GBR 1000 G.M.T. time signal in milliseconds
	MSF 60 kc/s 1429-1530 G.M.T.	Droitwich 200 kc/s 1030 G.M.T.	
1	−1·2	+4	−18·5
2	N.M.	+5	N.M.
3	N.M.	N.M.	N.M.
4	−1·2	+4	−22·3
5	−1·3	+5	−23·3
6	−1·2	+6	−25·2
7	−1·3	+6	−27·4
8 *	—	+1	—
9	−1·1	0	N.M.
10	−1·3	+1	N.M.
11	−1·2	+1	−32·9
12	−1·3	+1	−34·0
13	N.M.	+2	−36·2
14	−1·3	+1	−38·1
15	N.M.	+2	−39·0
16	N.M.	+2	−41·6
17	−1·0	+2	−42·7
18	−1·1	+1	−44·6
19	N.M.	+3	−46·0
20	−1·1	+2	−46·7
21	−1·1	+2	−49·3
22	−1·1	+1	−51·0
23	−1·1	−2	N.M.
24	−1·0	+2	N.M.
25	−1·0	+5	−54·3
26	−1·0	+6	−55·5
27	−1·0	+6	−57·1
28	−1·0	+6	N.M.
29	−1·1	+6	−59·9
30	−1·1	0	N.M.
31	−1·0	−1	N.M.

The values are based on astronomical data available on 1st February 1954.

The transmitter employed for the 60-kc/s signal is sometimes required for another service.

N.M. = Not Measured. * = No MSF Transmission.



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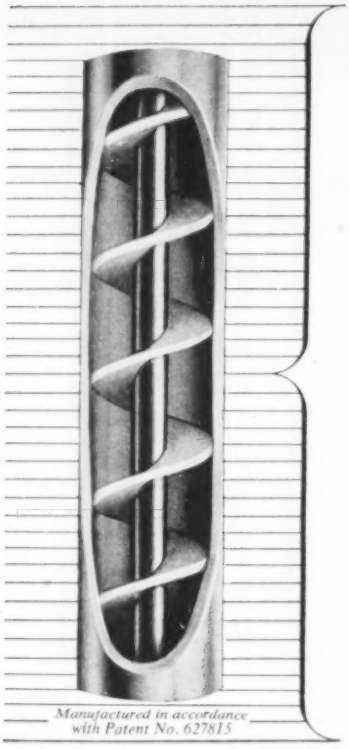

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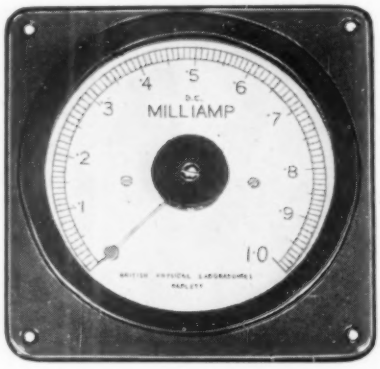

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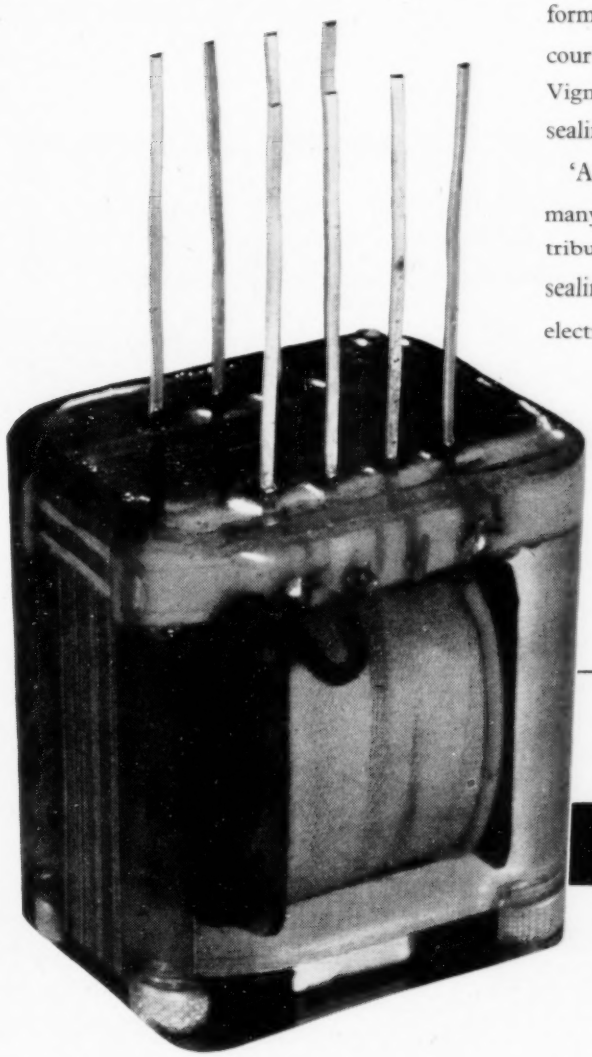

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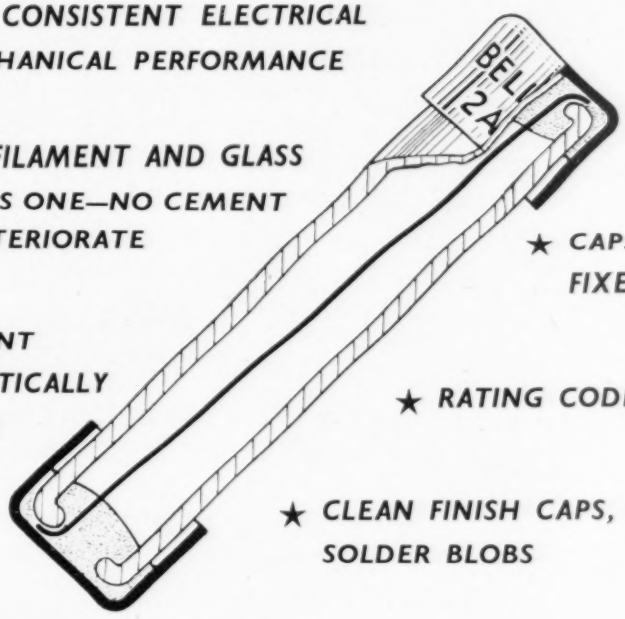

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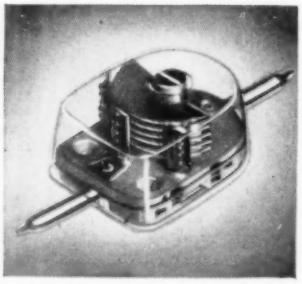

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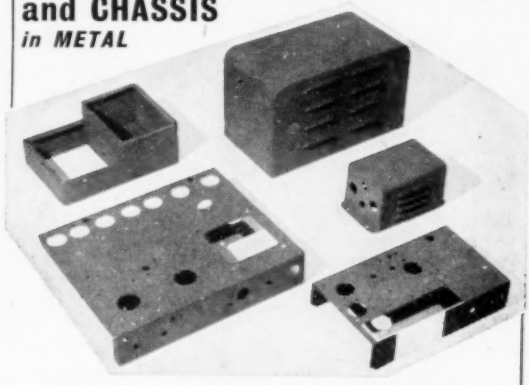

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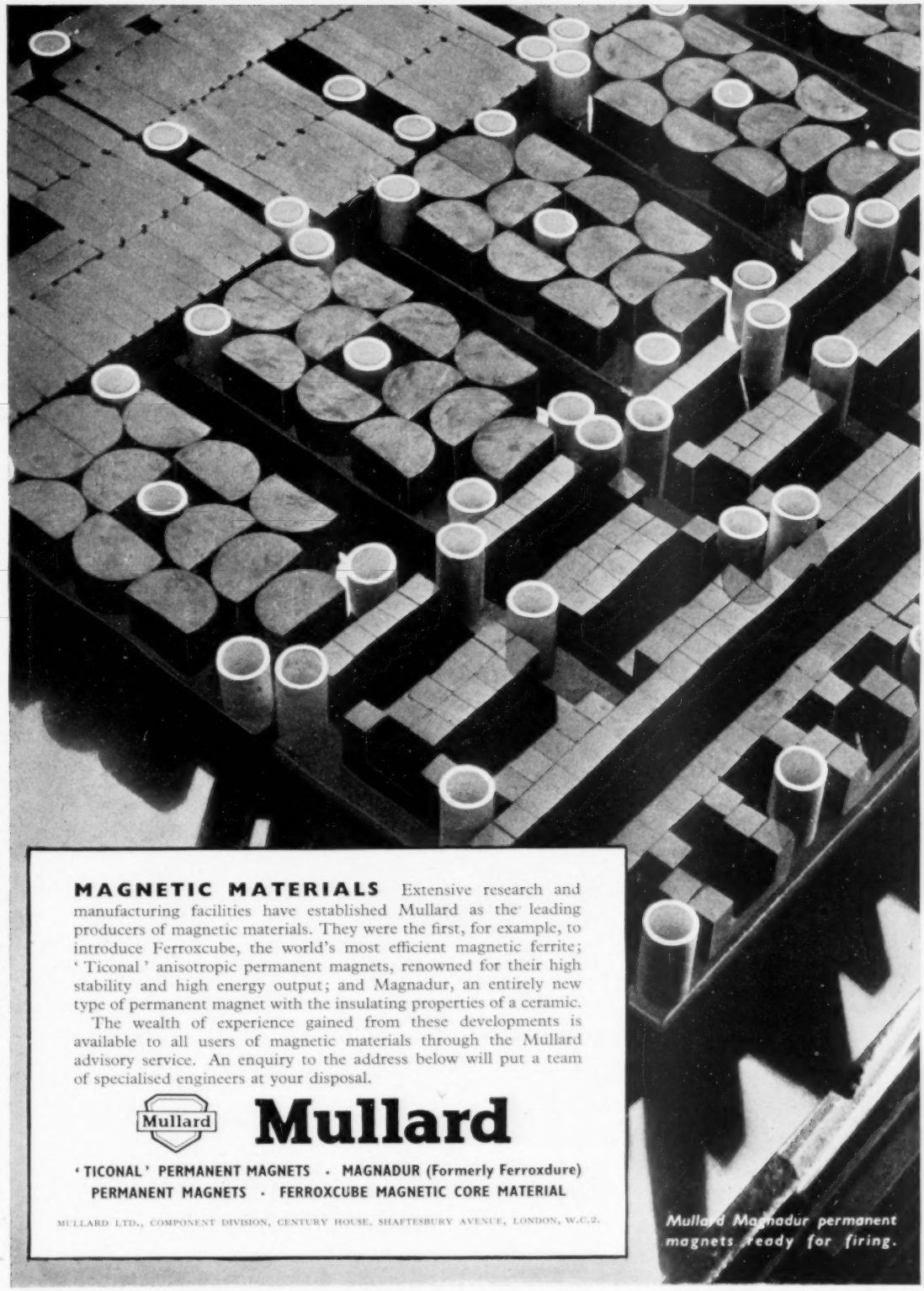

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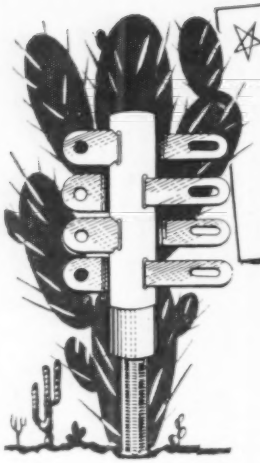

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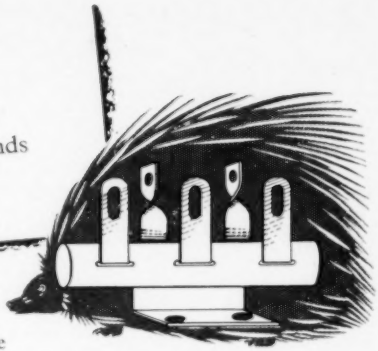

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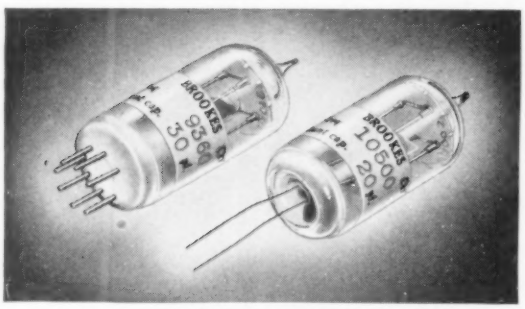

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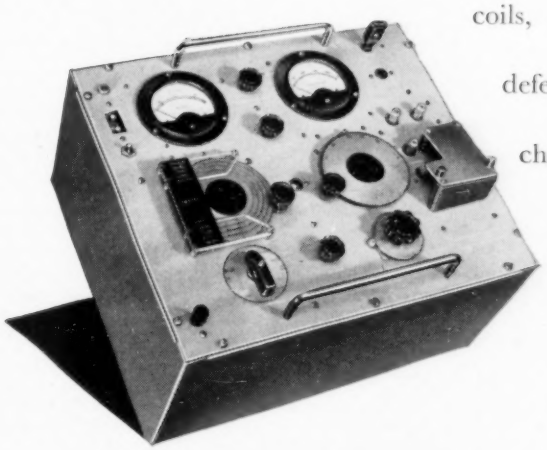

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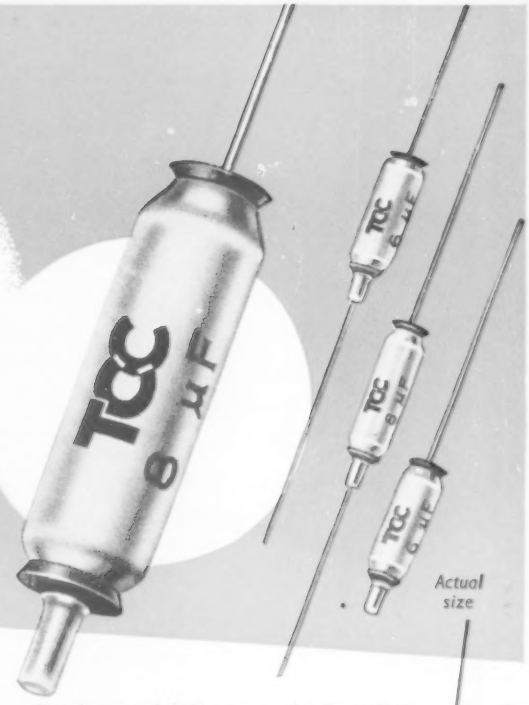

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8	6	.71	.2	CE69A
2	12	.64	.18	CE68B
4	12	.71	.2	CE69B
*8	15	.75	.26	CE67B
1	25	.64	.18	CE68C
2	25	.71	.2	CE69C
.5	50	.64	.18	CE68D
1	50	.71	.2	CE69D

* This condenser can only be supplied with wire terminations both ends.

T.C.C *Patent Nos. 578 487, 578 509, 587 072.*

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SPECIALISTS IN CONDENSERS SINCE 1906

Approved to ENTIRELY NEW Inter-Service Standards

A miniature range of Hunts Electrolytics approved to Inter-Service standards RCS 134 Issue 3 and RCL 134 Addendum Issue 2. Temperature range: −30° C to +70° C max. Please write for leaflet giving details of complete range.

Cap µF	Peak Working Volts at 70°C.	Max. R.M.S. Current at 50/c s (mA)	Dimensions (inches) L.	D.	List Number	Inter-Service Cat. Number
TYPE L31/i. PATTERN CE4. CLASS HI						
50	25	70	1 1/4	3/8	JB 53AKZ	Z145512
100	25	100	1 3/4	3/8	JB 54KZ	Z145514
1000	25	600	3	1	JB 57KZ	Z145520
25	50	60	1 1/4	3/8	JB102BKZ	Z145508
50	50	100	1 3/4	3/8	JB103KZ	Z145513
500	50	450	3	1	JB106AKZ	Z145519
8	150	60	1 1/4	3/8	JB153BKZ	Z145502
16	150	90	1 3/4	3/8	JB154KZ	Z145505
32	150	160	1 3/4	1/2	JB181KZ	Z145509
8	350	75	1 3/4	3/8	JB403KZ	Z145503
16	350	120	1 3/4	1/2	JB405KZ	Z145506
32	350	225	2	1	JB407AKZ	Z145510
4	450	50	1 3/4	3/8	JB552KZ	Z145501
8	450	100	1 3/4	1/2	JB553BKZ	Z145504
16	450	175	2	1	JB554AKZ	Z145507
32	450	275	3	1	JB555AKZ	Z145511
TYPE L32/1. PATTERN CE5 CLASS HI						
3000	25	1100	4 1/2	1 3/8	KB 62KZ	Z145557
1500	50	1000	4 1/2	1 3/8	KB111KZ	Z145555
60	350	350	2	1 3/8	KB430KZ	Z145552
100	350	450	3	1 3/8	KB411KZ	Z145554
32	450	275	3	1	KB555BKZ	Z145551
60	450	450	3	1 3/8	KB581KZ	Z145553
TYPE L32/3. PATTERN CE6. CLASS HI						
32+32	350	200	2	1 3/8	KB417KZ	Z145601
60+100	350	400	4 1/2	1 3/8	KB420KZ	Z145603
60+250	350	400	4 1/2	1 1/2	KB422KZ	Z145605
100+200	350	550	4 1/2	1 3/8	KB423KZ	Z145606
32+32	450	300	3	1 3/8	KB564AKZ	Z145602
60+100	450	550	4 1/2	1 3/8	KB565KZ	Z145604

REGISTERED TRADE MARK

HUNTS CAPACITORS

THE TRADE MARK OF RELIABILITY

All units are insulated by a Suflex sleeve and dimensions must be increased by 1/8" on the length and 1/16" on the diameter to allow for this sleeve.

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